Double two-photon correlated-spontaneous-emission lasers as bright sources of squeezed light

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We show that in a double two-photon correlated-spontaneous-emission laser, generation of squeezed light is compatible with atomic inversion by using the atomic coherence between two lower levels. The resulting device can yield, via resonant stimulated amplification, bright beams of squeezed light.

I. INTRODUCTION

The search for a bright source of squeezed light has been one of the strong motivations in the quest for lasers and masers based on two-photon transitions between levels of the same parity. Most of the work on these devices has centered around the possibility of intense and short-pulse generation and wide tunability,[1] as well as on the novel dynamical properties of these systems, associated with the fact that in this case we have an analogy with a first-order phase transition, instead of a second-order one as in the ordinary one-photon laser.[2,3] The possibility of using these devices for generating squeezed light stems from the work of Yuen,[4] who showed that an effective two-photon Hamiltonian (with terms quadratic in annihilation and creation operators) generates squeezed states of the radiation field. It was soon realized, however, that in lasers and masers the spontaneous-emission noise associated with an inverted system ends up destroying any possibility of obtaining squeezed light at steady state.[5] Only transient squeezing becomes then possible.[3] This is the reason why parametric amplifiers have been preferred as generators of squeezed light.[6,7] In these systems the atoms are far from saturation, due to the fact that the fields are of low intensity and off resonance with respect to the atomic transitions. Under these conditions the spontaneous-emission noise is negligible.[8] On the other hand, one usually gets only feeble sources of squeezed light in this way, as has been the case in recent experimental observations of this phenomenon.[9]

The search for intense sources of squeezed light is more timely than ever, in view of the far reaching potential applications. These are, among many others, in improving sensitivity limits of optical interferometers,[10] in spectroscopy with resolution below the natural linewidth,[11] or in noise quieting in active devices.[12] Compatibility of squeezing with stimulated emission and atomic inversion is therefore a highly desirable goal.

Interest in two-photon oscillators has recently been revived by the experimental demonstration of the continuous-wave operation of a two-photon micromaser.[13] New theoretical work has appeared, dealing with the differences between effective Hamiltonian and three-level models,[14] and with the role of atomic coherence.[15,16] In fact, it has been shown[16] that if the atoms are pumped into an appropriate superposition of the lasing states, quenching of spontaneous-emission noise may occur, thus leading to the simultaneous presence of squeezing and gain in a resonant process. The corresponding devices are called "correlated-emission-lasers" (CEL), and are presently under experimental investigation.[17]

In the present work, we show that contrary to common belief, it is possible to have population inversion *and* phase-noise squeezing in a laser. We show that appreciable intensity of squeezed light is produced, via stimulated emission, even when about 50% of squeezing in the phase is obtained, for the field inside the cavity (which corresponds to perfect squeezing outside[18]).

In the following section we define our model writing down the corresponding Hamiltonian, as well as the master equation for the reduced density matrix of the field in the linear approximation. In Sec. III, we derive a Fokker-Planck equation and a phase-locking condition, while in Sec. IV we show that squeezing in the phase is compatible with stimulated gain and even population inversion of the atomic system. Our conclusions are summarized in Sec. V. Details of the derivation of the master equation are given in Appendix A, while in Appendix B we present a microscopic theory of the degenerate parametric amplifier and compare it to the treatment based on an effective Hamiltonian approach.

II. MODEL

We consider a system of four-level atoms interacting with a single-mode field of frequency ν. The atoms (see Fig. 1) are pumped into a coherent superposition of states. The Hamiltonian for the system, in the rotating-wave approximation, is given by

$$H = H_0 + H_1 , \tag{2.1}$$

with

$$H_0 = \hbar\nu a^\dagger a + \sum_{i=a,b,c,d} \hbar\omega_i |i\rangle\langle i| , \tag{2.1a}$$

and

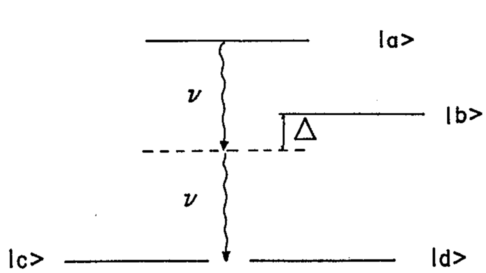

FIG. 1. Atomic levels relevant to the two-photon laser. The atoms are injected in a coherent superposition of levels $|a\rangle$ and $|c\rangle$, $|a\rangle$ and $|d\rangle$, and $|c\rangle$ and $|d\rangle$, but with no initial population in level $|b\rangle$. Levels $|c\rangle$ and $|d\rangle$ are considered to be almost degenerate. The field oscillates, with frequency ν. In this work, two-photon resonance is assumed, but the level $|b\rangle$ is detuned by Δ with respect to the one-photon resonance.

$$H_1=\hbar g(a|a\rangle\langle b|+a|b\rangle\langle c|+a|b\rangle\langle d|)+\text{H.c.} \quad (2.1\text{b})$$

Here a $(a^\dagger)$ is the field destruction (creation) operator, $\hbar\omega_i$ is the energy of level i and $|i\rangle$ are the atomic states $(i=a,b,c,d)$. We have assumed that only the $|a\rangle-|b\rangle$, $|b\rangle-|c\rangle$, and $|b\rangle-|d\rangle$ transitions are allowed with the corresponding atom-field coupling coefficient being equal to g. The operating frequency of the laser is given by ν.

Assuming that the atoms are injected into the cavity at a rate r_0 with initial population ρ_{ii} $(i=a,c,d)$ and initial coherences ρ_{ac}, ρ_{ad}, and ρ_{cd} [with $\rho_{ij}=\rho_{ji}^*$ and $\rho_{ij}=|\rho_{ij}|\exp(i\theta_{ij})$] we obtain the following master equation for the reduced density matrix of the field (for derivation see Appendix A):

$$\dot\rho=-\frac{\alpha}{2}[\rho_{aa}\mathcal{L}_1(aa^\dagger\rho-a^\dagger\rho a)+(\rho_{cc}\mathcal{L}_2+\rho_{dd}\mathcal{L}_3+\rho_{dc}\mathcal{L}_6\mathcal{L}_3+\rho_{cd}\mathcal{L}_6^*\mathcal{L}_2)(\rho a^\dagger a-a\rho a^\dagger)$$
$$+(\rho_{ca}\mathcal{L}_1\mathcal{L}_4+\rho_{da}\mathcal{L}_1\mathcal{L}_5)(aa\rho-a\rho a)+(\rho_{ca}\mathcal{L}_2\mathcal{L}_4+\rho_{da}\mathcal{L}_3\mathcal{L}_5)(\rho aa-a\rho a)]-\frac{\gamma}{2}(a^\dagger a\rho-a\rho a^\dagger)$$
$$-i(\Omega-\nu)a^\dagger a\rho+\text{H.c.}\ , \quad (2.2)$$

where $\mathcal{L}_i=\Gamma/(\Gamma-\Delta_i)$, with Γ being the atomic decay constant which, for simplicity, we have taken to be equal for all four levels, $\Delta_1=\omega_a-\omega_b-\nu$, $\Delta_2=\omega_b-\omega_c-\nu$, $\Delta_3=\omega_b-\omega_d-\nu$, $\Delta_4=\omega_a-\omega_c-2\nu$, $\Delta_5=\omega_a-\omega_d-2\nu$, and $\Delta_6=\omega_c-\omega_d$. The linear gain coefficient α is given by $\alpha=2rg^2/\Gamma^2$ and $\gamma=\Omega/2Q$ is the usual cavity loss, with Q being the cavity quality factor. Ω is the bare cavity eigenfrequency.

Equation (2.2) is similar to the corresponding equation for a two-photon three-level CEL [Eq. (1) of Ref. 16], but they have some substantial differences. Since here we do not have any initial excitation to level $|b\rangle$, there is no term linear in the coupling constant which has the form of an injected signal. Secondly, the absorption term [second in term in Eq. (2.2)] is phase dependent in the present case, due to the presence of ρ_{cd} and ρ_{dc}. As we show below the presence of these extra terms helps to extend the range of populations for which squeezing is still possible.

III. FOKKER-PLANCK EQUATION AND PHASE LOCKING

In order to study the phase noise, we convert Eq. (2.2) into an equivalent Fokker-Planck equation for the Glauber-Sudarshan P representation $P(\epsilon,\epsilon^*,t)$, in the usual way[19] $(a|\epsilon\rangle=\epsilon|\epsilon\rangle)$. We then get

$$\frac{\partial}{\partial t}P(\epsilon,\epsilon^*,t)=-\frac{\partial}{\partial\epsilon}(d_\epsilon P)-\frac{\partial}{\partial\epsilon^*}(d_{\epsilon^*}P)$$
$$+2\frac{\partial^2}{\partial\epsilon\,\partial\epsilon^*}(D_{\epsilon\epsilon^*}P)+\frac{\partial^2}{\partial\epsilon^2}(D_{\epsilon\epsilon}P)$$
$$+\frac{\partial^2}{\partial\epsilon^{*2}}(D_{\epsilon^*\epsilon^*}P)\ , \quad (3.1)$$

with

$$d_\epsilon=(d_{\epsilon^*})^*=R\epsilon+G^*\epsilon^*\ , \quad (3.1\text{a})$$

$$D_{\epsilon\epsilon}=(D_{\epsilon^*\epsilon^*})^*=\rho_{ac}\mathcal{L}_2^*\mathcal{L}_4^*+\rho_{ad}\mathcal{L}_3^*\mathcal{L}_5^*\ , \quad (3.1\text{b})$$

$$R=\frac{\alpha}{2}(\rho_{aa}\mathcal{L}_1-\rho_{cc}\mathcal{L}_2^*-\rho_{dd}\mathcal{L}_3^*-\rho_{cd}\mathcal{L}_6^*\mathcal{L}_3^*-\rho_{dc}\mathcal{L}_6\mathcal{L}_2^*)$$
$$-\frac{\gamma}{2}-i(\Omega-\nu)\ , \quad (3.1\text{c})$$

$$G=\frac{\alpha}{2}[\rho_{ca}(\mathcal{L}_1-\mathcal{L}_2)\mathcal{L}_4+\rho_{da}(\mathcal{L}_1-\mathcal{L}_3)\mathcal{L}_5]\ , \quad (3.1\text{d})$$

$$D_{\epsilon^*\epsilon}=\frac{\alpha}{8}\rho_{aa}(\mathcal{L}_1+\mathcal{L}_1^*)\ . \quad (3.1\text{e})$$

Introducing now polar coordinates as $\epsilon=re^{i\phi}$ we obtain the drift and diffusion coefficients associated with phase and amplitude variables ϕ and r. We get then[20]

$$d_r=r\,\text{Re}(R+Ge^{2i\phi})+O(1/r)\ , \quad (3.2)$$

$$d_\phi=\text{Im}(R-Ge^{2i\phi})+O(1/r^2)\ , \quad (3.3)$$

$$D_{rr}=\tfrac{1}{2}[D_{\epsilon^*\epsilon}+\text{Re}(D_{\epsilon^*\epsilon^*}e^{2i\phi})]\ , \quad (3.4)$$

$$D_{\phi\phi}=\frac{1}{r^2}[D_{\epsilon^*\epsilon}-\text{Re}(D_{\epsilon^*\epsilon^*}e^{2i\phi})]\ . \quad (3.5)$$

Comparison of (3.1) with the corresponding Fokker-Planck equation for the degenerate amplifier[6] shows the essential differences between the two processes. In the case of the parametric amplifier, one gets a Fokker-Planck equation like the one given by (3.1) but with $R=-\gamma_1$, $G^*=\kappa\epsilon/\gamma_2$, $D_{\epsilon\epsilon}=\kappa\epsilon/2\gamma_2$, $D_{\epsilon\epsilon^*}=0$, where γ_1 and γ_2 are the cavity losses for the pumping mode 1 and the signal mode 2, respectively, ϵ is the classical driving field, and κ is the coupling constant between the pump and the signal modes.

One should, however, notice that the derivation of the Fokker-Planck equation for the parametric amplifier is based on the effective Hamiltonian approach[6] which ignores the atomic fluctuations. In order to make a direct comparison with the present work we derive, in Appendix B, the effective Hamiltonian model starting from a microscopic atomic model of the problem. Then we show that atomic fluctuations can indeed be neglected in the limit of large detuning and for intensities far below saturation.

It is clear that the injected coherences ρ_{ac} and ρ_{ad} in the present model correspond to the field ϵ, and that our equations differ from those in Ref. 6 essentially because of the presence, in our case, of population-dependent terms, as well as the extra coherence ρ_{cd}. We notice, in particular, the presence in (3.4) and (3.5) of the term $D_{\epsilon^*\epsilon}$, which is proportional to ρ_{aa} [cf. Eq. (3.1e)], and tends to increase the diffusion coefficients. We show however that, by properly preparing the initial coherences, it is still possible to simultaneously obtain squeezing and stimulated emission gain.

We focus our attention on the special case when the levels $|c\rangle$ and $|d\rangle$ are almost degenerate ($\Delta_6 \ll \Gamma$), the two-photon resonance conditions $\Delta_4=\Delta_5=0$ are satisfied and the intermediate level $|b\rangle$ is detuned by an amount Δ such that $\Delta_2=\Delta_3=-\Delta_1=\Delta$. Under these conditions $\mathcal{L}_2=\mathcal{L}_3=\mathcal{L}_1^*=\mathcal{L}=\Gamma/(\Gamma-i\Delta)$ and $\mathcal{L}_4=\mathcal{L}_5=\mathcal{L}_6\approx 1$. We also consider $|\rho_{ca}|=\rho_{da}|$, $\rho_{cc}=\rho_{dd}$. With these simplifications we obtain

$$\langle\dot{\phi}\rangle=\langle d_\phi\rangle=-\frac{\alpha\Delta\Gamma}{2(\Gamma^2+\Delta^2)}\left\langle\rho_{aa}-2\rho_{cc}-2|\rho_{cd}|\left[\frac{\Gamma^2+\Delta^2}{\Delta^2}\right]^{1/2}\cos\theta_{cd}\sin\mu\right.$$
$$\left.-4|\rho_{ca}|\cos\left[\frac{\theta_{ca}+\theta_{da}}{2}+2\phi\right]\cos\left[\frac{\theta_{cd}}{2}\right]\right\rangle+\nu-\Omega+O(1/r^2)\,, \tag{3.6}$$

$$D_{\phi\phi}=\frac{\alpha}{4r^2}|\mathcal{L}|^2\left[\rho_{aa}-\frac{2|\rho_{ca}|}{|\mathcal{L}|}\cos\left[\frac{\theta_{ca}+\theta_{da}}{2}+2\phi+\mu\right]\cos\left[\frac{\theta_{cd}}{2}\right]\right]\,, \tag{3.7}$$

where $\mu=\tan^{-1}(\Delta/\Gamma)$, and we have used that $\theta_{ca}-\theta_{da}=\theta_{cd}$.

From (3.6) we get both a frequency pulling equation

$$\nu=\Omega+\frac{\alpha\Delta\Gamma}{2(\Delta^2+\Gamma^2)}\left[\rho_{aa}-2\rho_{cc}-2|\rho_{cd}|\left[\frac{\Gamma^2+\Delta^2}{\Delta^2}\right]^{1/2}\cos\theta_{cd}\sin\mu\right]\,, \tag{3.8}$$

and a phase-locking equation

$$\langle\dot{\phi}\rangle=\frac{2\alpha\Delta\Gamma}{\Gamma^2+\Delta^2}|\rho_{ca}|\cos\left[\frac{\theta_{cd}}{2}\right]\left\langle\cos\left[\frac{\theta_{ca}+\theta_{da}}{2}+2\phi\right]\right\rangle\,. \tag{3.9}$$

Phase-locking occurs for

$$\phi=\phi_0=-\frac{\theta_{ca}+\theta_{da}}{4}-(\tfrac{1}{4}\pm\tfrac{1}{2})\pi\,\mathrm{sgn}\left[\Delta\cos\frac{\theta_{cd}}{2}\right]\,. \tag{3.10}$$

The existence of two stable solutions of the locked phase is a peculiar feature of this two-photon device. The same kind of result was obtained in Ref. 16, and it can be understood heuristically in the following way. Since the injected coherences ρ_{ab}, ρ_{bc}, and ρ_{bd} are zero in this case, one has to consider the field produced by the two-photon polarization ρ_{ac} and ρ_{ad}, which add up when $|\rho_{ac}|=|\rho_{ad}|$ to a two-photon polarization, proportional to

$$|\rho_{ac}|\cos[(\theta_{ac}-\theta_{ad})/2]\exp[i(\theta_{ac}+\theta_{ad})/2]\,.$$

This polarization is, however, a source for the *square* of the electric field, since a two-photon process is involved. This means that the usual locking equation is now satisfied by 2ϕ, which can have only one stable value. Since this value is defined modulo 2π, it turns out that ϕ is given modulo π, yielding therefore two stable values for the phase of the field. A nonlinear treatment of this model may then reveal which of the two values is more probable, through an analysis of the overall probability distribution for the field. This nonlinear approach will be presented elsewhere.

IV. LASING AND SQUEEZING

Under the phase-locking condition (3.10) the diffusion coefficient $D_{\phi\phi}$ becomes

$$D_{\phi\phi}|_{\phi=\phi_0}\equiv D(\phi_0)$$
$$=\frac{\alpha}{4r^2}|\mathcal{L}|^2\left[\rho_{aa}-2\left|\rho_{ca}\frac{\Delta}{\Gamma}\cos\left[\frac{\theta_{cd}}{2}\right]\right|\right]\,. \tag{4.1}$$

On the other hand, from (3.2) we have

$$\langle \dot{r} \rangle = \langle d_r \rangle = \left\langle \frac{\alpha r}{2} \mathrm{Re}\{\rho_{aa}\mathcal{L}_1 - \rho_{cc}\mathcal{L}_2 - \rho_{dd}\mathcal{L}_3 - \rho_{dc}\mathcal{L}_6\mathcal{L}_3 - \rho_{cd}\mathcal{L}_6^*\mathcal{L}_2 + e^{2i\phi}[\rho_{ca}(\mathcal{L}_1 - \mathcal{L}_2)\mathcal{L}_4 + \rho_{da}(\mathcal{L}_1 - \mathcal{L}_3)\mathcal{L}_5]\} - \frac{\gamma r}{2} + O(1/r) \right\rangle , \tag{4.2}$$

which yields the linear gain. This expression is composed of three types of terms. Those proportional to the populations represent the usual contributions to the gain in a laser, and yield a positive contribution when population inversion holds. The terms proportional to ρ_{ca} and ρ_{da} stand for an extra contribution to the gain, due to the injected polarization associated with the coherent superposition of level $|a\rangle$ with levels $|c\rangle$ and $|d\rangle$. They lead to a klystron-type gain (not stimulated-emission-type gain), and were already present in Ref. 16. The terms proportional to ρ_{cd} are new, and are responsible for lasing without inversion in a degenerate quantum-beat laser.[21]

Under the detuning conditions described earlier, the expression for the net gain G, defined by

$$\langle \dot{r} \rangle = \tfrac{1}{2}\langle Gr \rangle , \tag{4.3}$$

simplifies considerably and we obtain

$$G = \alpha|\mathcal{L}|^2 \left[\rho_{aa} - \rho_{cc} - \rho_{dd} - 2|\rho_{dc}|\cos\theta_{dc} + 2\frac{\Delta}{\Gamma}|\rho_{ca}|\cos\left[\theta_{ca} + 2\phi - \frac{\pi}{2}\right] + 2\frac{\Delta}{\Gamma}|\rho_{da}|\cos\left[\theta_{da} + 2\phi - \frac{\pi}{2}\right]\right] - \gamma . \tag{4.4}$$

Under the phase-locking condition (3.10) and with $|\rho_{ca}| = |\rho_{da}|$, $\rho_{cc} = \rho_{dd} = |\rho_{cd}|$, G becomes

$$G = \alpha|\mathcal{L}|^2 \left[\rho_{aa} - 4\rho_{cc}\cos^2\left[\frac{\theta_{cd}}{2}\right] + 4\left|\frac{\Delta}{\Gamma}\rho_{ca}\cos\left[\frac{\theta_{cd}}{2}\right]\right|\right] - \gamma . \tag{4.5}$$

This expression can be optimized with respect to θ_{cd}, so that the maximum gain is obtained for

$$\left|\cos\left[\frac{\theta_{cd}}{2}\right]\right| = \frac{|\Delta|}{2\Gamma}\frac{|\rho_{ca}|}{\rho_{cc}} . \tag{4.6}$$

When this condition is satisfied

$$G = \alpha\rho_{aa} - \gamma , \tag{4.7}$$

and

$$D(\phi_0) = \frac{\alpha}{4\bar{n}}|\mathcal{L}|^2\rho_{aa}\left[1 - \left[\frac{\Delta}{\Gamma}\right]^2\right] , \tag{4.8}$$

where we have used that $|\rho_{ac}|^2 = \rho_{aa}\rho_{cc}$ (pure state), and $r^2 \approx \bar{n}$, the average number of photons.

Expression (4.7) has a remarkable simplicity: the gain depends only on the population of the upper level and it does not depend on the initial population density of the lower levels $|c\rangle$ and $|d\rangle$. It is therefore possible to have a net stimulated emission gain and, hence, lasing even in the noninversion regime. This result has its physical origin in the quantum interference which is brought about by the initial preparation of the atomic system in a coherent superposition of levels.[21] This interference eliminates the absorption of radiation by the atoms while still allowing emission. This effect is present even if one does not introduce the coherence between states $|c\rangle$ and $|d\rangle$: Just ρ_{ac} is sufficient to suppress absorption.[16] We show below, however, that the extra coherence leads to a rather unexpected effect, viz., it extends the region for which upper-level population and squeezing are compatible. The extra coherence brings up yet another unexpected result: the linear gain does not depend on the detuning [cf. (4.7)].

In order to calculate the amount of squeezing obtainable in this system, we recall that the phase uncertainty in steady state is given by[20]

$$\langle(\delta\phi)^2\rangle = \frac{1}{4\bar{n}} + \langle :(\delta\phi)^2: \rangle , \tag{4.9}$$

where the first term is the contribution due to shot noise arising from vacuum fluctuations and the second term is due to spontaneous-emission noise. This last term is related to the drift coefficient d_ϕ and the diffusion coefficient $D_{\phi\phi}$ [Eqs. (3.3) and (3.5)] by[20]

$$\langle :(\delta\phi)^2: \rangle = D(\phi_0)|\partial d_\phi / \partial\phi|^{-1}_{\phi=\phi_0} , \tag{4.10}$$

where the derivative of the drift coefficient is evaluated at the phase-locking point. It follows then from (3.9) and (4.6) that

$$\left.\frac{\partial d_\phi}{\partial\phi}\right|_{\phi=\phi_0} = -\frac{2\alpha\Delta^2}{\Gamma^2+\Delta^2}\rho_{aa} , \tag{4.11}$$

and, therefore, from Eqs. (4.8), (4.9), and (4.11),

$$\langle(\delta\phi)^2\rangle = \frac{1}{8\bar{n}}\left[1 + \frac{\Gamma^2}{\Delta^2}\right] . \tag{4.12}$$

We see that $\langle(\delta\phi)^2\rangle$ gets smaller than the shot-noise contribution $(4\bar{n})^{-1}$ so long as $\Delta > \Gamma$ [from Eq. (48) we see that this condition corresponds to a negative phase-diffusion coefficient in the P representation]. On the other hand, it follows from (4.6) that

$$\Gamma^2/\Delta^2 \gtrsim \frac{|\rho_{ca}|^2}{4\rho_{cc}^2} = \frac{\rho_{aa}}{4\rho_{cc}} = \frac{\rho_{aa}}{2(1-\rho_{aa})} . \tag{4.13}$$

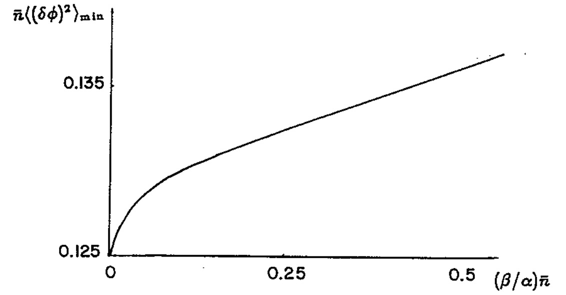

FIG. 2. Plot of $\bar{n}\langle(\delta\phi)^2\rangle_{min}$ vs the nonlinear intensity parameter $(\beta/\alpha)\bar{n}$ for $(\Delta/\Gamma)=\delta=10$.

Maximum squeezing is obtained when the equality holds, so that

$$\langle(\delta\phi)^2\rangle_{min}=\frac{1}{8\bar{n}}\left[1+\frac{\rho_{aa}}{2(1-\rho_{aa})}\right] . \quad (4.14)$$

For $\rho_{aa}=0$ the maximum amount of squeezing is attained. We notice, however, that in the present case squeezing persists up to $\rho_{aa}=\frac{2}{3}$. This result should be contrasted to those of Ref. 16, where no squeezing was found if there was population inversion. We see, therefore, that the addition of a fourth level indeed extends the range of upper-state population for which squeezing is still present and helps to conciliate population inversion and squeezing.

Ideally, one would like to get close to 50% squeezing inside the cavity, which means that ρ_{aa} should be much smaller than unity. It is important to notice, however, that even in this case one may still have a bright source of squeezed light because the gain is independent of the detuning [see Eq. (4.7)], and can be larger than the gain of a usual two-photon laser with incoherent pumping to the upper level.

Indeed, it has been shown by a nonlinear analysis of the three-level system[22] that in the above-threshold regime $\langle(\delta\phi)^2\rangle$ increases when $\bar{n}$, the number of photons, increases. For a particular choice of parameters one obtains

$$\langle(\delta\phi)^2\rangle_{min}=\frac{1}{8\bar{n}}\left[1+\frac{[3(\beta/\alpha)\bar{n}]^{1/2}}{|\Delta/\Gamma|}\right] . \quad (4.15)$$

Here $\beta/\alpha=4g^2/\Gamma^2$ is the nonlinearity parameter in lasers.[19] In Fig. 2 we plot $\bar{n}\langle(\delta\phi)^2\rangle_{min}$ versus the nonlinear intensity parameter $(\beta/\alpha)\bar{n}$ for $|\Delta/\Gamma|=10$. The curve indicates that $\langle(\delta\phi)^2\rangle_{min}$ increases slowly with the increase of $\bar{n}$. A typical value for β/α is 10^{-7}. For example, one still has 48% squeezing even when the laser intensity reaches $\bar{n}=0.1(\alpha/\beta)$ which corresponds to 10^6 photons in the cavity. Thus a bright output is compatible with a large amount of intracavity squeezing (very near to the ideal 50%).

V. DISCUSSION AND CONCLUSION

We have shown that it is possible to reconcile two-photon lasing with squeezed-light generation, so long as the injected atoms are prepared in a coherent superposition of states. The resulting device is based on a combination of two recently developed ideas: the correlated-emission laser[15,16] and the laser without inversion.[21] It has been proven in previous works[5] that a usual two-photon laser with incoherent pumping cannot generate squeezed field output due to noise contributions from spontaneous-emission fluctuation and vacuum fluctuation (shot noise). In the two-photon laser, the noise is a sum of these two noise sources. Therefore, even after quenching the spontaneous-emission noise, we are still left with the shot noise, i.e., we still cannot reach noise squeezing. In order to obtain noise squeezing, one needs to introduce atomic coherence into the system. This is how the two-photon CEL can achieve noise squeezing below the vacuum noise level. In degenerate parametric oscillators (DPO), interaction between the pump field and the nonlinear material is far off resonant, and creates a small amount of upper-level population and a full but small coherence between the upper and ground levels $\rho_{aa}\ll\rho_{ac}=\sqrt{\rho_{aa}\rho_{cc}}$. Because of the small upper-level population, the spontaneous emission can be neglected, but this in itself would only lead to noise quenching, not to noise squeezing. It is the atomic coherence that leads to squeezing in the DPO. In this sense (that atomic coherence leads to noise squeezing) the DPO and CEL are similar. However, the gain, which stems from both ρ_{aa} and ρ_{ac}, is quite different for the DPO and the CEL. Because of the far off resonance in the DPO, the gain is very small. In the CEL operating on two-photon resonance, both the population and the coherence can be large compared to the DPO case. Consequently, the gain in the CEL is much higher than that in the DPO, provided the other conditions are the same. It is precisely in this sense that we call the two-photon CEL a bright source of squeezed radiation.

While the possibility of a bright squeezed-light source was already present in Ref. 16, here we have shown that the inclusion of degenerate lower levels increases the range of atomic populations for which squeezing is possible, to the point where even population inversion ($\rho_{aa}>\frac{1}{2}$) is permitted. We speculate that the addition of more coherently prepared lower levels might help to increase this range even further. Nonlinear terms might, however, reduce this range, as has been shown for the three-level case in Ref. 22. A nonlinear analysis of our model is planned to be presented elsewhere.

ACKNOWLEDGMENTS

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APPENDIX A: DERIVATION OF EQ. (2.2)

In the interaction picture, the Hamiltonian (2.1) can be rewritten as

$$V=(\Omega-\nu)a^\dagger a+g(ae^{i\Delta_1 t}|a\rangle\langle b|+ae^{i\Delta_2 t}|b\rangle\langle c| + ae^{i\Delta_3 t}|b\rangle\langle d|+\text{H.c.}) , \quad (A1)$$

where $\Delta_1=\omega_a-\omega_b-\nu$, $\Delta_2=\omega_b-\omega_c-\nu$, and $\Delta_3=\omega_b$

$-\omega_d - \nu$ with ν being the field frequency. An equation of motion for the reduced density operator of the field ρ is obtained by taking a trace over atoms which leads to

$$\dot{\rho} = -i(\Omega - \nu)[a^\dagger a, \rho] - ([V_{ab}, \tilde{\rho}_{ba}] + [V_{bc}, \tilde{\rho}_{cb}] + [V_{bd}, \tilde{\rho}_{db}] + \text{H.c.}) . \quad \text{(A2)}$$

In Eq. (A2), $\tilde{\rho}$ denotes the full atom-field density operator and $V_{ab} = ga\exp(i\Delta_1 t)$, $V_{bc} = ga\exp(i\Delta_2 t)$, and $V_{bd} = ga\exp(i\Delta_3 t)$. The atomic matrix elements $\tilde{\rho}_{ba}$, $\tilde{\rho}_{cb}$, and $\tilde{\rho}_{db}$ can be evaluated to the first order in the coupling constant g by solving the equations of motion for the corresponding matrix elements using a perturbation theory. For example,

$$\dot{\tilde{\rho}}_{ba} = -i(V_{ba}\tilde{\rho}_{aa} + V_{bc}\tilde{\rho}_{ca} + V_{bd}\tilde{\rho}_{da} - \tilde{\rho}_{bb}V_{ba}) , \quad \text{(A3)}$$

whose first-order solution is given by

$$\tilde{\rho}_{ba}(t) = -i\int_{t_0}^{t} d\tau \{ [V_{ba}(\tau)\rho_{aa} + V_{bc}(\tau)\rho_{ca} + V_{bd}(\tau)\rho_{da}]\rho(t_0) - \rho(t_0)\rho_{bb}V_{ba}(\tau)\} , \quad \text{(A4)}$$

where t_0 is the time of injection of the atom. If we sum over all the atoms which are injected at a rate r, we obtain

$$\tilde{\rho}_{ba}(t) = -ir\int_{-\infty}^{t} dt_0 e^{-\Gamma(t-t_0)} \int_{t_0}^{t} d\tau \{ [V_{ba}(\tau)\rho_{aa} + V_{bc}(\tau)\rho_{ca} + V_{bd}(\tau)\rho_{da}]\rho(t) - \rho(t)\rho_{bb}V_{ba}\} . \quad \text{(A5)}$$

In writing Eq. (A5), we have assumed that the atomic lifetime is $1/\Gamma$ (which is assumed to be the same for all atomic levels) and we have replaced $\rho(t_0)$ by $\rho(t)$ in the usual Markovian approximation. In a similar manner

$$\tilde{\rho}_{cb}(t) = -ir\int_{-\infty}^{t} dt_0 e^{-\Gamma(t-t_0)} \int_{t_0}^{t} d\tau \{ V_{cb}(\tau)\rho_{bb}\rho(t) - \rho(t)[\rho_{ca}V_{ab}(\tau) + \rho_{cc}V_{cb}(\tau) + \rho_{cd}V_{db}(\tau)]\} , \quad \text{(A6)}$$

$$\tilde{\rho}_{db}(t) = -ir\int_{-\infty}^{t} dt_0 e^{-\Gamma(t-t_0)} \int_{t_0}^{t} d\tau \{ V_{db}(\tau)\rho_{bb}\rho(t) - \rho(t)[\rho_{da}V_{ab}(\tau) + \rho_{dc}V_{cb}(\tau) + \rho_{dd}V_{db}(\tau)]\} . \quad \text{(A7)}$$

The integrations in Eqs. (A5)–(A7) are simple and straightforward. After carrying out these integrations, the expressions for $\tilde{\rho}_{ba}$, $\tilde{\rho}_{cb}$, and $\tilde{\rho}_{db}$ can be substituted in Eq. (A2). The resulting equation is given by Eq. (2.2) of the text.

APPENDIX B: MICROSCOPIC MODEL OF THE DEGENERATE PARAMETRIC AMPLIFIER

We show in this appendix how the degenerate parametric amplifier Hamiltonian can be obtained from a model which takes into account the atomic fluctuations. For the four-wave-mixing problem, this question has been considered in Ref. 8. We consider for simplicity a two-level atom which interacts with both the pump and the signal modes. The Hamiltonian describing the system is taken to be

$$H = \hbar\omega a_1^\dagger a_1 + 2\hbar\omega a_2^\dagger a_2 + \frac{\hbar\omega_0}{2}\sigma_3 + \hbar g_1(a_1^2\sigma_+ + a_1^{\dagger 2}\sigma_-) + \hbar g_2(a_2\sigma_+ + a_2^\dagger\sigma_-) + (\epsilon a_2^\dagger + \epsilon^\dagger a_2) , \quad \text{(B1)}$$

where $\sigma_3 = \sigma_a - \sigma_b$, $\sigma_a = |a\rangle\langle a|$, $\sigma_b = |b\rangle\langle b|$, $\sigma_+ = |a\rangle\langle b|$, $\sigma_- = (\sigma_+)^\dagger$, $|a\rangle$ and $|b\rangle$ are the upper and lower atomic states, a_i and $a_i^\dagger$ are the annihilation and creation operators for the signal $(i=1)$ and pump $(i=2)$ modes, and ϵ is a classical driving field for the pump mode.

In order to simplify our treatment, we have used an effective Hamiltonian for the two-photon process; it can be obtained from a three-level atomic model by adiabatically eliminating the intermediate state, when the level is highly detuned from the one-photon resonance, so that its population can be neglected.[23] We assume this to be the case.

From the above Hamiltonian, and after adding the usual loss terms and their respective fluctuation forces, we get the following equations of motion for the atomic and field operators:

$$\dot{a}_1 = -i\omega a_1 - \frac{\gamma_1}{2}a_1 - 2ig_1 a_1^\dagger\sigma_- + F_1 , \quad \text{(B2)}$$

$$\dot{a}_2 = -2i\omega a_2 - \frac{\gamma_2}{2}a_2 - ig_2\sigma_- + \epsilon + F_2 , \quad \text{(B3)}$$

$$\dot{\sigma}_3 = -\Gamma(\sigma_3 - \sigma_3^{\text{eq}}) - 2ig_1(\sigma_+ a_1^2 - a_1^{\dagger 2}\sigma_-) - 2ig_2(\sigma_+ a_2 - a_2^\dagger\sigma_-) + F_3 , \quad \text{(B4)}$$

$$\dot{\sigma}_- = -i\omega_0\sigma_- - \Gamma\sigma_- + ig_1\sigma_3 a_1^2 + ig_2\sigma_3 a_2 + F_- , \quad \text{(B5)}$$

where for simplicity we have assumed that all atomic decay rates coincide, and σ_3^{eq} is a diagonal matrix which yields the equilibrium values, at zero field, of the atomic populations. The Langevin noise operators on the right-hand side of the above equations are specified by their first and second moments. Thus, for F_1 and F_2 we have[19,24] $(i,j=1,2)$

$$\langle F_i(t)\rangle = 0, \quad \langle F_i^\dagger(t)F_j(t')\rangle = \delta_{ij}\gamma_i n_{\text{th}}\delta(t-t') ,$$
$$\langle F_i(t)F_j(t')\rangle = 0 , \quad \text{(B6)}$$
$$\langle F_i(t)F_j^\dagger(t')\rangle = \delta_{ij}\gamma_i(n_{\text{th}}+1)\delta(t-t') ,$$

where n_{th} is the number of thermal photons in the cavity, while for the atomic noise operators we have[19,24]

$$\langle F_-^{\dagger}(t)F_-(t')\rangle=\Gamma\langle\sigma_a(t)+\sigma_a^{\rm eq}\rangle\delta(t-t')\ , \quad \text{(B7)}$$

$$\langle F_3(t)F_3(t')\rangle =\Gamma\langle\sigma_a(t)+\sigma_b(t)+\sigma_a^{\rm eq}+\sigma_b^{\rm eq}\rangle\delta(t-t')\ , \quad \text{(B8)}$$

$$\langle F_-(t)F_3(t')\rangle=\Gamma\langle\sigma_-(t)\rangle\delta(t-t')\ , \quad \text{(B9)}$$

among the nonvanishing correlation functions.

We remove now the high-frequency terms from the above equations by introducing the new operators

$$a_1=\tilde{a}_1e^{-i\omega t},\ \ a_2=\tilde{a}_2e^{-2i\omega t},\ \ \tilde{\sigma}_-=\tilde{\sigma}_-e^{-2i\omega t}\ , \quad \text{(B10)}$$

and the corresponding fluctuation forces

$$F_1=\tilde{F}_1e^{-i\omega t},\ \ F_2=\tilde{F}_2e^{-2i\omega t},\ \ F_-=\tilde{F}_-e^{-2i\omega t}\ , \quad \text{(B11)}$$

as well as the new driving field,

$$\epsilon=\tilde{\epsilon}e^{-i2\omega t}\ . \quad \text{(B12)}$$

The new operators satisfy the equations of motion

$$\dot{\tilde{a}}_1=-\frac{\gamma_1}{2}\tilde{a}_1-2ig_1\tilde{a}_1^{\dagger}\tilde{\sigma}_-+\tilde{F}_1\ , \quad \text{(B13)}$$

$$\dot{\tilde{a}}_2=-\frac{\gamma_2}{2}\tilde{a}_2-ig_2\tilde{\sigma}_-+\tilde{\epsilon}+\tilde{F}_2\ , \quad \text{(B14)}$$

$$\dot{\sigma}_3=-\Gamma(\sigma_3-\sigma_3^{\rm eq})-2ig_1(\tilde{\sigma}_+\tilde{a}_1^2-\tilde{a}_1^{\dagger 2}\tilde{\sigma}_-) -2ig_2(\tilde{\sigma}_+\tilde{a}_2-\tilde{a}_2^{\dagger}\tilde{\sigma}_-)+F_3\ , \quad \text{(B15)}$$

$$\dot{\tilde{\sigma}}_-=-i(\omega_0-2\omega)\tilde{\sigma}_--\Gamma\tilde{\sigma}_-+ig_1\sigma_3\tilde{a}_1^2+ig_2\sigma_3\tilde{a}_2+\tilde{F}_-\ . \quad \text{(B16)}$$

We now transform these equations into c-number equations, in the usual way,[19,24] choosing the normal ordering $a^{\dagger},\sigma_+,\sigma_3,\sigma_-,a$. We thus get four c-number Langevin equations of the variables $\epsilon_1,\epsilon_2,\tilde{\sigma}_-,\sigma_3$. We then get

$$\dot{\epsilon}_1=-\frac{\gamma_1}{2}\epsilon_1-2ig_1\epsilon_1^*\Sigma_-+\mathcal{F}_1\ , \quad \text{(B17)}$$

$$\dot{\epsilon}_2=-\frac{\gamma_2}{2}\epsilon_2-ig_2\Sigma_-+\tilde{\epsilon}+\mathcal{F}_2\ , \quad \text{(B18)}$$

$$\dot{\Sigma}_3=-\Gamma(\Sigma_3-\Sigma_3^{\rm eq})-2ig_1(\Sigma_+\epsilon_1^2-\epsilon_1^{*2}\Sigma_-) -2ig_2(\Sigma_+\epsilon_2-\epsilon_2^*\Sigma_-)+\mathcal{F}_3\ , \quad \text{(B19)}$$

$$\dot{\Sigma}_-=-i(\omega_0-2\omega)\Sigma_--\Gamma\Sigma_-+ig_1\Sigma_3\epsilon_1^2+ig_2\Sigma_3\epsilon_2+\mathcal{F}_-\ . \quad \text{(B20)}$$

The functions $\mathcal{F}$ on the right-hand side of the above equations must satisfy the relations

$$\langle\mathcal{F}_k(t)\rangle=0\ , \quad \text{(B21)}$$

$$\langle\mathcal{F}_k(t)\mathcal{F}_l(t')\rangle=\langle 2D_{kl}\rangle\delta(t-t')\ , \quad \text{(B22)}$$

where the diffusion coefficients D_{kl} are determined by the requirement that the equation of motion for the second moments are identical to the corresponding operator equations. This implies that some of the diffusion coefficients change, when one goes from the operator to the c-number description.[19,24,25]

Thus, for instance, from the operator equation (B16), we get

$$\frac{d}{dt}\langle\tilde{\sigma}_-(t)\tilde{\sigma}_-(t)\rangle=-[i(\omega_0-2\omega)+\Gamma/2]\langle\tilde{\sigma}_-(t)\tilde{\sigma}_-(t)\rangle+ig_1\langle\sigma_3\tilde{\sigma}_-\tilde{a}_1^2+\tilde{\sigma}_-\sigma_3\tilde{a}_1^2\rangle+ig_2\langle\tilde{\sigma}_-\sigma_3\tilde{a}_2+\sigma_3\tilde{\sigma}_-\tilde{a}_2\rangle +\langle\tilde{\sigma}_-\tilde{F}_-\rangle+\langle\tilde{F}_-\tilde{\sigma}_-\rangle\ . \quad \text{(B23)}$$

Bringing all the terms into normal order, and using that $\langle\tilde{\sigma}_-\tilde{F}_-\rangle=\langle\tilde{F}_-\tilde{\sigma}_-\rangle=0$, we get

$$\frac{d}{dt}\langle\tilde{\sigma}_-(t)\tilde{\sigma}_-(t)\rangle=-[i(\omega_0-2\omega)+\Gamma/2]\langle\tilde{\sigma}_-(t)\tilde{\sigma}_-(t)\rangle+2ig_1\langle\sigma_3\tilde{\sigma}_-\tilde{a}_1^2\rangle+2ig_1\langle\tilde{\sigma}_-\tilde{a}_1^2\rangle+2ig_2\langle\sigma_3\tilde{\sigma}_-\tilde{a}_2\rangle +2ig_2\langle\tilde{\sigma}_-\tilde{a}_2\rangle\ . \quad \text{(B24)}$$

On the other hand, from (B20) we obtain

$$\frac{d}{dt}\langle\Sigma_-(t)\Sigma_-(t)\rangle=-[i(\omega_0-2\omega)+\Gamma/2]\langle\Sigma_-(t)\Sigma_-(t)\rangle+2ig_1\langle\Sigma_3\Sigma_-\epsilon_1^2\rangle+2ig_2\langle\Sigma_3\Sigma_-\epsilon_2\rangle+\langle 2D_{\Sigma_-\Sigma_-}\rangle\ . \quad \text{(B25)}$$

Requiring that (B24) and (B25) should coincide, we find that

$$2D_{\Sigma_-\Sigma_-}=2ig_1\Sigma_-\epsilon_1^2+2ig_2\Sigma_-\epsilon_2\ . \quad \text{(B26)}$$

In the same way we find that the diffusion coefficients of the fields not change, while the other relevant atomic diffusion coefficients are given by

$$2D_{\Sigma_+\Sigma_-}=\Gamma(\Sigma_a+\Sigma_a^{\rm eq})\ , \quad \text{(B27)}$$

$$2D_{\Sigma_+\Sigma_3}=-\Gamma\Sigma_+\ , \quad \text{(B28)}$$

$$2D_{\Sigma_3\Sigma_3}=\Gamma(\Sigma_a+\Sigma_b+\Sigma_a^{\rm eq}+\Sigma_b^{\rm eq})-4ig_1(\Sigma_+\epsilon_1^2-\epsilon_1^{*2}\Sigma_-) -4ig_2(\Sigma_+\epsilon_2-\epsilon_2^*\Sigma_-)\ . \quad \text{(B29)}$$

We now proceed to the adiabatic elimination of the atomic variables, assuming that $\Gamma \gg \gamma_1, \gamma_2$. As a first step, we set the time derivative of Σ_- equal to zero in (B20), so that

$$\Sigma_- = i\frac{g_1\epsilon_1^2 + g_2\epsilon_2}{1+i\delta}\frac{\Sigma_3}{\Gamma} + \frac{1}{1+i\delta}\frac{\mathcal{F}_-}{\Gamma}, \quad \text{(B30)}$$

where we have defined

$$\delta = (\omega_0 - 2\omega)/\Gamma . \quad \text{(B31)}$$

Substituting this result into the equations for ϵ_1, ϵ_2, and Σ_3 yields

$$\dot\epsilon_1 = -\frac{\gamma_1}{2}\epsilon_1 + \Gamma(\Omega_1|\epsilon_1|^2\epsilon_1 + \Omega_1\epsilon_1^*\epsilon_2)\Sigma_3 + \mathcal{F}_1 - \frac{2ig_1\epsilon_1^*}{1+i\delta}\frac{\mathcal{F}_-}{\Gamma}, \quad \text{(B32)}$$

$$\dot\epsilon_2 = -\frac{\gamma_2}{2}\epsilon_2 + \frac{\Gamma}{2}(\Omega\epsilon_1^2 + \Omega_2\epsilon_2)\Sigma_3 + \mathcal{E} + \mathcal{F}_2 - \frac{ig_2}{1+i\delta}\frac{\mathcal{F}_-}{\Gamma}, \quad \text{(B33)}$$

$$\dot\Sigma_3 = -\Gamma(\Sigma_3 - \Sigma_3^{\rm eq}) - \Gamma[2\Omega_{1R}|\epsilon_1|^4 + 2\Omega_{2R}|\epsilon_2|^2 + 4\Omega_R \mathrm{Re}(\epsilon_1^{*2}\epsilon_2)]\Sigma_3 + \mathcal{F}_3 + 2i\frac{g_1\epsilon_1^{*2} + g_2\epsilon_2^*}{1+i\delta}\frac{\mathcal{F}_-}{\Gamma} - \frac{2i(g_1\epsilon_1^2 + g_2\epsilon_2)}{1-i\delta}\frac{\mathcal{F}_+}{\Gamma}, \quad \text{(B34)}$$

where we have defined

$$\Omega_k = \frac{2g_k^2/\Gamma^2}{1+i\delta} = \Omega_{kR} + i\Omega_{kI}, \quad k = 1,2 \quad \text{(B35)}$$

$$\Omega = \frac{2g_1g_2/\Gamma^2}{1+i\delta} = \Omega_R + i\Omega_I . \quad \text{(B36)}$$

We eliminate now the variable Σ_3, setting the time derivative equal to zero in (B34) and solving for Σ_3

$$\Sigma_3 = \left[\Sigma_3^{\rm eq} + \mathcal{F}_3/\Gamma + 2i\frac{g_1\epsilon_1^{*2} + g_2\epsilon_2^*}{1+i\delta}\frac{\mathcal{F}_-}{\Gamma^2} - 2i\frac{g_1\epsilon_1^2 + g_2\epsilon_2}{1-i\delta}\frac{\mathcal{F}_+}{\Gamma^2}\right] \Big/ D , \quad \text{(B37)}$$

where

$$D = 1 + 2\Omega_{1R}|\epsilon_1|^4 + 2\Omega_{2R}|\epsilon_2|^2 + 4\Omega_R \mathrm{Re}(\epsilon_1^{*2}\epsilon_2) . \quad \text{(B38)}$$

We now insert (B37) into (B32) and (B33), so that

$$\dot\epsilon_1 = -\frac{\gamma_1}{2}\epsilon_1 + (\Omega_1|\epsilon_1|^2\epsilon_1 + \Omega\epsilon_1^*\epsilon_2)\Sigma_3^{\rm eq}\Gamma/D + \mathcal{F}_1' , \quad \text{(B39)}$$

$$\dot\epsilon_2 = -\frac{\gamma_2}{2}\epsilon_2 + (\Omega\epsilon_1^2 + \Omega_2\epsilon_2)\Sigma_3^{\rm eq}\Gamma/2D + \mathcal{E} + \mathcal{F}_2' , \quad \text{(B40)}$$

where the noise forces $\mathcal{F}_1'$ and $\mathcal{F}_2'$ are given by

$$\mathcal{F}_1' = \mathcal{F}_1 + \frac{(\Omega_1|\epsilon_1|^2\epsilon_1 + \Omega\epsilon_1^*\epsilon_2)\mathcal{F}_3}{D} + \frac{2i}{1+i\delta}\left[\frac{\Omega_1|\epsilon_1|^2\epsilon_1 + \Omega\epsilon_1^*\epsilon_2}{D}(g_1\epsilon_1^{*2} + g_2\epsilon_2^*) - g_1\epsilon_1^*\right]\frac{\mathcal{F}_-}{\Gamma} - \frac{2i}{1-i\delta}\frac{\Omega_1|\epsilon_1|^2\epsilon_1 + \Omega\epsilon_1^*\epsilon_2}{D}(g_1\epsilon_1^2 + g_2\epsilon_2)\frac{\mathcal{F}_+}{\Gamma}, \quad \text{(B41)}$$

$$\mathcal{F}_2' = \mathcal{F}_2 + (\Omega\epsilon_1^2 + \Omega_2\epsilon_2)\frac{\mathcal{F}_3}{2D} + \frac{i}{1+i\delta}\left[\frac{\Omega\epsilon_1^2 + \Omega_2\epsilon_2}{D}(g_1\epsilon_1^{*2} + g_2\epsilon_2^*) - g_2\right]\frac{\mathcal{F}_-}{\Gamma} - \frac{i}{1-i\delta}\frac{\Omega\epsilon_1^2 + \Omega_2\epsilon_2}{D}(g_1\epsilon_1^2 + g_2\epsilon_2)\frac{\mathcal{F}_+}{\Gamma} . \quad \text{(B42)}$$

From (B41) and (B42), and using (B26)–(B29) one can now calculate the correlation functions of $\mathcal{F}_1'$ and $\mathcal{F}_2'$. For our purposes, however, it is sufficient to notice that, if the following conditions are met:

$$1 \ll \delta, \quad |\Omega_1\epsilon_1^4| \ll 1, \quad |\Omega_2\epsilon_2^2| \ll 1 , \quad \text{(B43)}$$

which correspond to large detuning and small one- and two-photon Rabi frequencies (compared with Γ), then (B39) and (B40) can be written as

$$\dot\epsilon_1 = -\frac{\gamma_1}{2}\epsilon_1 - i\Delta\omega_1[1 + O(1/\delta)]\epsilon_1 - 2i\kappa[1 + O(1/\delta)]\epsilon_1^*\epsilon_2 + \mathcal{F}_1' , \quad \text{(B44)}$$

$$\dot\epsilon_2 = -\frac{\gamma_2}{2}\epsilon_2 - i\Delta\omega_2[1 + O(1/\delta)]\epsilon_2 - i\kappa[1 + O(1/\delta)]\epsilon_1^2 + \mathcal{E} + \mathcal{F}_2' . \quad \text{(B45)}$$

where

$$\Delta\omega_1 = 2g_1^2|\epsilon_1|^2\Sigma_3^{\rm eq}/\Gamma\delta , \quad \text{(B46)}$$

$$\Delta\omega_2 = g_2^2\Sigma_3^{\rm eq}/\Gamma\delta , \quad \text{(B47)}$$

and

$$\kappa = g_1g_2\Sigma_3^{\rm eq}/\Gamma\delta . \quad \text{(B48)}$$

We notice that $\Delta\omega_1$ and $\Delta\omega_2$ correspond to frequency-pulling terms, associated with the dispersive part of the atomic refractive index (there are also contributions from the absorptive part, which change the values of γ_1 and γ_2 and are of higher order in $1/\delta$). These terms can be absorbed into redefined frequencies and absorption coefficients for modes 1 and 2. The remaining contribution (excluding for the moment the fluctuation forces) can be obtained from the Hamiltonian

$$H=\hbar\omega a_1^\dagger a_1+2\hbar\omega a_2^\dagger a_2+\hbar\kappa(a_1^{\dagger 2}a_2+a_2^\dagger a_1^2)+(\epsilon a_2^\dagger+\epsilon^* a_2)\ , \tag{B49}$$

which is precisely the degenerate optical parametric Hamiltonian (with frequency-pulling terms ignored).

As for the fluctuation forces, the terms added to $\mathcal{F}_1$ and $\mathcal{F}_2$ in (B41) and (B42) represent additional contribution to noise stemming from the atomic operators. It is clear that only for sufficiently large detunings and low intensities may these contributions be ignored. One gets then, in this limit, the Langevin equation for the degenerate amplifier.

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