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Classical Behavior with Small Quantum Numbers: The Physics of Ramsey Interferometry of Rydberg Atoms

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In Ramsey atomic interferometry, a superposition of atomic states is produced by a mechanism completely equivalent (for experimental purposes) to interaction with a classical field. Since this property holds, in the case of Rydberg atoms, for temperatures close to absolute zero and field intensities of the order of a single photon, the question arises as to why the quantum nature of the field can be neglected. We model the passage of an atom through a Ramsey zone and show that, in order to explain the phenomenon, correlation properties between three subsystems and strong cavity dissipation turn out to be the essential physical ingredients leading to classical behavior. [S0031-9007(99)09302-3]

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Microwave cavities are extensively used in experiments designed to access fundamental issues concerning the interaction of atoms and electromagnetic field modes in the context of cavity quantum electrodynamics [1]. In Ramsey atomic interferometry [2], in particular, they are known to generate quantum superpositions of atomic states as if the field inside them were of a classical nature. Two cavities separated by an intermediate region are filled with fields oscillating with phase coherence so that atomic transition probability amplitudes undergo quantum superpositions observed as interference (Ramsey) fringes. When this situation holds for temperatures close to absolute zero and field intensities of the order of a single photon, one may ask to what extent the quantum nature of the field can be neglected and how can this be theoretically modeled from basic quantum theory.

In contrast to high-quality-factor cavities, in which photons dissipate at sufficiently low rates, the cavities used in such interferometric devices must be continuously pumped by an external source in order to make up for the relatively short photon lifetimes, if a stationary state is to be maintained in them. In this respect the cavity mode must be considered as a damped system. Open quantum systems have been the subject of renewed interest in many areas including quantum optics [3]. The interaction with a large external reservoir provides, within the standard quantum mechanical framework, one way of accounting for effective nonunitary dissipative subsystem dynamics [4], a characteristic of which is to act on the coherence properties of the subsystem states. Quantum coherence can be thereby destroyed [5], as is observed in most of the macroscopic world and theoretically expected also in a mesoscopic scale. Such decoherence processes often take place at very short time scales, so that they can be studied, in some models at least, by means of a perturbative short-time expansion for the coherence loss, which can be measured, e.g., by the so-called linear entropy (or idempotency defect) [6]. The progressive decoherence (due to the relatively small dissipation in the high-quality-factor cavity) of mesoscopic quantum superspositions of field coherent states ("Schrödinger cat states") has recently been experimentally observed [7], after producing the field superposition states by interaction with atoms. In this experiment, atomic states are steered, both before and after interacting with the high-quality cavity, by fields in low-quality cavities (Ramsey zones) which, as mentioned above, behave classically as interferometric tools.

In order to model the behavior of the field in such low-Q cavities, we consider three subsystems, namely, the atom, described as a two level system (a pair of nearly resonant Rydberg states), the relevant cavity mode, pumped by an external classical source, and a reservoir (heat bath) responsible for the dissipative properties of the cavity mode. It is not intended here to discuss general aspects of the dynamics of three interacting subsystems, but these turn out to be the minimum essential ingredients required to model the functioning of a weakly excited, low-Q cavity in the interferometric device. When an incoming atom, in the upper or lower relevant state, with velocity of the order of $v = 400$ m/s, moves through the cavity for an effective path length of about $l = 0.5$ cm, it takes a time of $T = l/v \simeq 10\ \mu$s to induce a $\pi/2$ pulse in its state space (both states being finally equally populated). This time is of the same order of magnitude as the atom-field coupling constant (Rabi frequency) $\Omega/2\pi \simeq 10$ kHz, as determined by cavity geometry and atomic structure (Rydberg circular states [7]). Atom-field entanglement, however, is suppressed. As shown in detail below from first principles, the field dissipation γ (of some MHz, i.e., quality factor $Q \sim 2 \times 10^3$ for resonant field and atomic transition frequencies $\omega_f = \omega_a$ of tens of GHz) accounts for this observed coherent atomic evolution.

We consider the fully quantized time-dependent Hamiltonian

$$H = H(t) = H_a + H_F(t) + H_{\rm int} = H_0(t) + H_{\rm int}, \quad (1)$$

where the atom, treated as a two level system, is described by

$$H_a = \tfrac{1}{2}\hbar\omega_a\sigma_z, \quad (2)$$

and the field, coupled to the reservoir responsible for its damping, is modeled as

$$\begin{aligned} H_F(t) &= H_{fb} + \hbar(F^* e^{i\omega t} a + F e^{-i\omega t} a^\dagger) \\ &= \hbar\omega_f a^\dagger a + \sum_k \hbar g_k (a^\dagger b_k + a b_k^\dagger) \\ &\quad + \sum_k \hbar\omega_k b_k^\dagger b_k + \hbar(F^* e^{i\omega t} a + F e^{-i\omega t} a^\dagger), \end{aligned} \quad (3)$$

which includes, in addition, the pumping by an external source of amplitude F tuned at near resonance (last term). As usual, $a^\dagger$ and a are the field (boson) creation and annihilation operators, $b_k^\dagger$ and b_k are the corresponding operators for the reservoir, taken as a large collection of harmonic oscillators, and σ_z is the standard Pauli matrix. In the absence of the source, the dissipative dynamics of a single mode of the electromagnetic field in a cavity is frequently modeled by the Hamiltonian H_{fb} [8]. In the long-time limit the dissipative dynamics can be described by a master equation in the Born-Markov approximation [8,9], whereby the relaxation of the field follows a standard exponential decay law. By including the external source and using more refined estimates for the dissipative dynamics [10], one may expect to obtain qualitatively equivalent final results, since the classical-field effect we seek is contained in the stationary dynamics of (3).

The atom-field coupling will be treated in the dipole and rotating wave approximations (RWAs) as

$$H_{\rm int} = \hbar\Omega(\sigma^\dagger a + \sigma a^\dagger), \quad (4)$$

where σ stands for the Pauli matrix $\frac{1}{2}(\sigma_x - i\sigma_y)$. In fact, the above interaction has proved adequate in many applications of the Jaynes-Cummings model in quantum optics [11]. Later, we will comment on effects arising from antiresonant terms. The state of the complete system is fully described by the density operator

$$\rho(t) = U(t)\rho(0)U^\dagger(t), \quad (5)$$

where $U(t)$ is the propagator for the total Hamiltonian (1) and $\rho(0)$ is the initial condition at $t = 0$, taken as the time at which the atom enters the cavity.

The role of the field damping on atomic coherence properties can be assessed, even in the absence of the external source F, by examining perturbatively the atomic linear entropy

$$\delta_a(t) = 1 - {\rm tr}_a \rho_a(t)^2, \quad (6)$$

where $\rho_a(t) = {\rm tr}_{fb}\rho(t)$ is the atomic reduced density. This quantity, which is initially zero for a pure, decorrelated atomic initial state, becomes positive as $\rho_a(t)$ loses its initial purity, as can be most conveniently calculated in an interaction picture defined as follows. The total Hamiltonian (1) is split into a free part consisting of $\hbar(\frac{1}{2}\omega_a\sigma_z + \omega_f a^\dagger a)$ to which we add a term $\hbar\omega_0 \sum_k b_k^\dagger b_k$ with $\omega_0 \simeq \omega_f \simeq \omega_a$; this renders the remaining interaction part time independent, containing the Hamiltonian (4), the field-bath interaction, and also a counterterm $\hbar\sum_k(\omega_k - \omega_0) b_k^\dagger b_k$. Following [6], the power series expansion of $\delta_a(t)$ for short times, with the initial condition $\rho(0) = \rho_a(0) \otimes \rho_f(0) \otimes \rho_b(0) = |+\rangle\langle+| \otimes |0\rangle\langle 0| \otimes |\{0\}_k\rangle\langle\{0\}_k|$ which describes an atom initially in the excited state while the field and the reservoir are in their respective ground states [12], yields up to fourth order in time

$$\delta_a(t) = \left[1 - \frac{1}{12}\left(\sum_k g_k^2\right)t^2\right]2\Omega^2 t^2 - \frac{8}{3}\Omega^4 t^4. \quad (7)$$

The term $2\Omega^2 t^2$ gives the usual short-time scale for atom-field decoherence [6]. It is eventually counterbalanced by the last term, and is *quenched* by the relaxation due to the coupling to the reservoir. This holds for times up to where the leading terms of (7) have a maximum, at $t_m = \sqrt{6}/\tilde{\gamma}$, with $\tilde{\gamma}^2 \equiv \sum_k g_k^2$, and even at this time limit the atomic decoherence as measured by $\delta_a(t_m) \simeq 6(\Omega/\tilde{\gamma})^2$ is small for $\Omega \ll \tilde{\gamma}$.

Alternatively, one can exploit the smallness of the atom-field coupling constant and use a perturbation expansion in powers of Ω in order to study the atomic decoherence. In this case, a straight interaction picture is used for $H_{\rm int}$ [Eq. (4)] still with no external source, so that restrictions related to its time dependence are removed. Here, in order to collect all powers of Ω up to fourth order, the field-reservoir sector H_{fb} is diagonalized as $a^\dagger = \sum_\nu \xi_{0\nu} v_\nu^\dagger$ with $\sum_\nu |\xi_{0\nu}|^2 = 1$, where the $v_\nu^\dagger$ is the νth field-bath eigenmode creation operator of frequency ω_ν. For the same initial condition as above, the result for $\delta_a(t)$ is

$$\delta_a(t) = 2\Omega^2 \sum_\nu |\xi_{0\nu}|^2 \left(\frac{\sin \frac{1}{2}(\omega_a - \omega_\nu)t}{\frac{1}{2}(\omega_a - \omega_\nu)} \right)^2 - \left(\frac{\Omega}{\gamma}\right)^4 f(x), \tag{8}$$

where $x = \gamma t$. As regards the dominant term, expanding the sine function for short times one recovers the leading behavior of (7) with $\sum_k g_k^2$ replaced by the mean-square linewidth $\sum_\nu |\xi_{0\nu}|^2 (\omega_a - \omega_\nu)^2$. Although Lorentzians do not possess this second moment, we can use them for qualitative purposes, in the continuum limit and for longer times, introducing a peak position $\omega_L \simeq \omega_a$ and a width $\gamma \ll \omega_L$. Extending then the integration towards negative frequencies, Eq. (8) becomes $\delta_a(t) \simeq 4(\Omega/\gamma)^2(\gamma t + e^{-\gamma t} - 1)$, where the contribution γt arises from the pole on the real axis at ω_a. This latter expression remains small even for times $T \simeq 1/\Omega$ for which $\delta_a(T) \simeq 4(\Omega/\gamma)$, if the field damping is suitably large ($\Omega \ll \gamma$). No substantial change comes from the correction of highest order in Ω in Eq. (8), which is given in the continuum limit by $f(x) = 2(6x^2 - 14x + 8xe^{-x} - 16e^{-x} + 5e^{-2x} + 11)$ and reduces for short times to that appearing in Eq. (7). It follows that the adopted initial atomic state $|+\rangle\langle+|$ remains very nearly a pure state after a time $1/\Omega$, which is the time scale for this state to rotate a fourth of a cycle in the atomic state space (a "$\pi/2$ pulse") when the external source is turned on. As indicated by the smallness of $\delta_a(t)$, this rotation is moreover nearly unitary, with little coherence loss.

A complete summation of the full series in powers of Ω, with the external source included, can be carried out by means of the normal modes $v_\nu^\dagger$, in the sense that an exact solution for the interaction picture operators $a^\dagger(t)$ and $a(t)$ can be obtained, leading eventually to the form

$$H_{\rm int}(t) = \hbar\Omega e^{-i\omega_a t}\sigma \left[i\int_0^t ds\, P^*(t-s)F^*(s) + P^*(t)a^\dagger + \sum_k r_k^*(t) b_k^\dagger \right] + \text{H.c.} \tag{9}$$

for the interaction Hamiltonian. In this expression, $P(t) = \sum_\nu |\xi_{0\nu}|^2 e^{-i\omega_\nu t}$ and $r_k(t) = \sum_\nu \xi_{0\nu}^* \xi_{k\nu} e^{-i\omega_\nu t}$ are effective couplings of the atom to the cavity mode and to the bath oscillators, respectively, $\xi_{k\nu}$ being the amplitude of $b_k^\dagger$ in the basis $v_\nu^\dagger$. Taking into account the unitarity of the transformation to normal modes, for very short times $P(t) \sim 1$, $r_k(t) \sim 0$ and the only significant term in the square brackets is the second one. In this time range the dynamics is therefore nearly of resonant Jaynes-Cummings-type, which tends to entangle the atom and the field on a time scale Ω^{-1} [6]. Equations (7) and (8), however, indicate that this does not, in fact, occur. In order to analyze this result in terms of Eq. (9), note that, for times longer than the ones just considered, a new dynamic regime sets in, in which the evolution of the field and that of the atom are progressively *decoupled*, as a result of a decrease of $P(t)$. This dynamic transition takes place on a scale set by the field relaxation time γ^{-1}. In fact, closed, approximate expressions for the effective couplings can be obtained within a single-pole Weisskopf-Wigner approximation to $a^\dagger(t)$ [8] which reads

$$P(t) \simeq e^{-(i\omega_f + \gamma)t}, \qquad r_k(t) \simeq \frac{g_k e^{-i\omega_k t}}{(\omega_k - \omega_f) + i\gamma}\left(1 - e^{i(\omega_k - \omega_f)t} e^{-\gamma t}\right). \tag{10}$$

Using these expressions, the object in the square brackets in Eq. (9) corresponds to the solution of the Heisenberg-Langevin equation for Markovian field dissipation, with decay constant $\gamma = \pi D(\omega_f)|g(\omega_f)|^2$, which results in the limit of reservoir oscillators having a continuous frequency distribution with number density $D(\omega)$. The last term of this object couples the atom to the bath oscillators with effective constants $g_k' \equiv (\omega_k - \omega_f + i\gamma)^{-1}\Omega g_k$. Within the Born-Markov approximation this corresponds, at resonance $\omega_a = \omega_f$, to an atomic damping constant $\gamma' \equiv \pi D(\omega_a)|g'(\omega_a)|^2 = (\Omega/\gamma)\Omega$. This atomic damping can be neglected when $\Omega \ll \gamma$ for interaction times which are not too long, in the sense that $\gamma' t \ll 1 \ll \gamma t$. The cavity dissipation thereby washes out the atomic decoherence which would otherwise take place in the scale Ω^{-1} due to atom-field entanglement. As a result, if the interaction time is limited to the range $t \ll 1/\gamma'$, there is no time for appreciable coherence loss of the atomic state, as its evolution is governed mainly by the first term in (9).

In the Schrödinger picture, the resulting evolution of the atomic reduced density $\rho_a(t)$ is thus described by a unitary Hamiltonian of the familiar form

$$H_R(t) = H_a + \hbar|\Delta|(e^{-i(\omega t-\phi)}\sigma^\dagger + \text{H.c.}) = H_a + \hbar|\Delta|[\sigma_x \cos(\omega t - \phi) + \sigma_y \sin(\omega t - \phi)], \tag{11}$$

where $\Delta \equiv \Omega F/(\omega - \omega_f + i\gamma) \equiv |\Delta|e^{i\phi}$, describing a classical field rotating around the quantization axis with the source frequency ω and phase ϕ [2]. Interference fringes are obtained by varying ω across $\omega_a = \omega_f$ over a range of a few kHz, where

$$|\Delta| = \Omega \frac{|F|}{\gamma} = \Omega\sqrt{\bar{n}}, \qquad |\omega - \omega_f| \ll \gamma,$$

where $\bar{n} = (|F|/\gamma)^2$ is the average photon number. In a Born-Markov approximation, this corresponds to the intensity of the stationary coherent state solution of the externally driven, damped cavity field Hamiltonian $H_F(t)$ [Eq. (3)] [13]. Hence, for transit time T, a $\pi/2$ pulse $|\Delta|T = \pi/4$ *does* require $\bar{n} \simeq 1$, and higher photon numbers therefore would need faster atoms. On the other hand, substantially higher-quality factors (longer photon lifetimes γ^{-1}) would lead to the merging of γ^{-1} and Ω^{-1}, thus helping atom-field correlations to survive long enough so as to induce decoherence as observed in superconducting cavities [7].

As a final comment, the effect of antiresonant terms in the interaction Hamiltonian (4) can be estimated as follows. An antiresonant term $\hbar\Omega'\sigma a$ in (9) adds to the coefficient of σ in the atomic Hamiltonian (11), a dominant contribution of the form $\hbar|\Delta'|e^{-i(\omega t-\phi)}$, with $|\Delta'| \equiv (\Omega'/\Omega)|\Delta|$. For Ω' of the order of Ω, this simply shifts the resonance frequency ω_a by an amount of the order of $|\Delta'|^2/\omega_a$ [2], which can be neglected for $|\Delta|$ of some kHz.

In conclusion, we have shown how classical behavior with small quantum numbers is possible. Specifically, a relatively strong damping of one among three correlated subsystems can provide for the quantum-fluctuation-free behavior of the Ramsey zone at zero temperature.

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