# Measurement of a negative value for the Wigner function of radiation

G. Nogues,[1] A. Rauschenbeutel,[1] S. Osnaghi,[1] P. Bertet,[1] M. Brune,[1] J. M. Raimond,[1] S. Haroche,[1] L. G. Lutterbach,[2] and L. Davidovich[2]

[1]*Laboratoire Kastler Brossel,* Département de Physique de l'Ecole Normale Supérieure, 24 rue Lhomond, F-75231 Paris Cedex 05, France*

[2]*Instituto de Fisica, Universidade Federal do Rio de Janeiro, Caixa Postale 68528, 21945-970 Rio de Janeiro, RJ, Brazil*

(Received 2 December 1999; published 11 October 2000)

Following a proposal by two of us [L. G. Lutterbach and L. Davidovich, Phys. Rev. Lett. **78**, 2547 (1997)], we have measured the Wigner function at the origin of phase space for a single photon field. Its value is negative, exhibiting the nonclassical nature of this state. The experiment is based on the absorption-free detection of the microwave field stored in a superconducting cavity [G. Nogues *et al.*, Nature (London) **400**, 239 (1999)]. Extension to a measurement of the Wigner function over the complete phase space is discussed.

PACS number(s): 03.65.−w, 42.50.Dv, 42.50.Ar

Since the invention of the laser, it became clear that the statistical properties of light may be quite different from the ones of the ''classical'' sources [1]. Considerable theoretical and experimental efforts have been devoted since then to the production and characterization of ''nonclassical'' states of light, such as antibunched, sub-Poissonian [2] or squeezed [3] states.

For a single-mode field, the statistical properties of light are vividly displayed by quasi-probability distributions in the two-dimensional phase space. Its axes correspond to the real and imaginary parts of the dimensionless complex field amplitude $\alpha$. Among these distributions, the Wigner function $W(\alpha)$ [4] has been particularly studied. It is well-suited for the calculation of the expectation values of field operators expressed in the symmetrical ordering [5]. The noncommutation of the photon annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$, which reveals the quantum nature of the light field, implies that $W$, which is real and bounded, may be negative, in contrast to a classical probability distribution. Proper choice of normalization [6] leads to $-2 \leqslant W \leqslant 2$.

For coherent states, thermal fields and even squeezed states, $W$ is positive. It thus behaves as a classical probability distribution. For coherent fields, $W$ is a Gaussian of width 1, with a maximum value of 2. In particular, Fig. 1(a) shows the Wigner function $W_0$ for the vacuum $|0\rangle$. For a thermal field, $W$ is also a Gaussian with a larger width and a reduced peak value. Finally, for a squeezed state, $W$ is a Gaussian with different widths along two orthogonal directions.

The observation of negative values of $W$ can thus be considered as a severe test of the nonclassical nature of a single-mode light field. Such negative values are expected for instance, for the Fock states $|n\rangle$ (with $n>0$). Figure 1(b) presents the Wigner function $W_1$ for the single-photon Fock state $|1\rangle$. It exhibits a dip at the origin, with a minimum value of $-2$. Negative values for $W$ are also expected for quantum superpositions of two coherent fields, the so-called

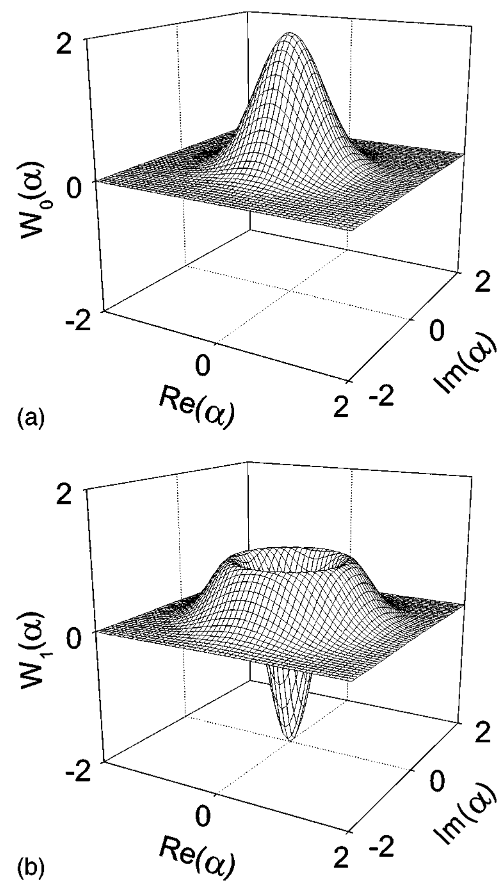


FIG. 1. Representations of Wigner distributions in phase space. (a) Vacuum field. (b) Single photon field, exhibiting clearly a negative value at origin.

''Schrödinger cat states'' [7,8]. An experimental measurement of negative $W$ values for these highly nonclassical states and of their evolution under field relaxation would give an extremely useful insight into the phenomenon of decoherence.

Optical quantum tomography [9] makes it possible to measure $W$. The field to be measured beats with a local oscillator in a homodyne arrangement. Measurements with

*Unité mixte de recherche, Ecole Normale Supérieure, Université Pierre et Marie Curie et CNRS (UMR8552).

 

many different phases of the local oscillator and an inversion algorithm allow one to reconstruct $W$. This has been performed up to now only for positive Wigner functions of coherent or squeezed states [10]. Other schemes have been proposed to determine $W$. For a state with density matrix $\hat{\rho}$, $W$ may be written as [6]

$$W(\alpha)=2\mathrm{Tr}[\hat{D}^{-1}(\alpha)\hat{\rho}\hat{D}(\alpha)\hat{P}]. \tag{1}$$

Here, $\hat{P}=\exp(i\pi\hat{a}^{\dagger}\hat{a})$ is the parity operator $(\hat{P}|n\rangle=(-1)^{n}|n\rangle)$, and $\hat{D}(\alpha)=\exp(\alpha\hat{a}^{\dagger}-\alpha^{*}\hat{a})$ is the displacement operator. When applied to $|0\rangle$, $\hat{D}(\alpha)$ produces a coherent state $|\alpha\rangle$ (an eigenstate of $\hat{a}$ with the eigenvalue $\alpha$). $W$ appears thus as twice the expectation value of $\hat{P}$ in the displaced field state $\hat{\rho}(\alpha)=\hat{D}^{-1}(\alpha)\hat{\rho}\hat{D}(\alpha)$. In particular, at the origin, $W(0)$ is twice the average of $\hat{P}$ in the original state $\hat{\rho}$. $W$ can also be expressed in terms of the photon-number probabilities $\hat{\rho}(\alpha)_{nn}=\langle n|\hat{\rho}|\alpha||n\rangle$ corresponding to the displaced density operator. From Eq. (1), one obtains, tracing in the Fock-state basis:

$$W(\alpha)=2\sum_{n}(-1)^{n}\hat{\rho}(\alpha)_{nn}. \tag{2}$$

In particular, $W_n(0)=2(-1)^n$, where $W_n$ is the Wigner function for the $|n\rangle$ Fock state. It is negative for all odd $n$-values.

Equation (2) has led to several proposals to measure $W$ by photon counting [11]. Another recent proposal for measuring $W$ for a cavity mode was based on Eq. (1). It uses as probes two-level atoms interacting dispersively with the field [12]. The dispersive interaction leads to a phase shift of the atomic wave function proportional to the photon number. This shift can be revealed by a Ramsey atomic interferometer [13]. Under proper conditions, the final state of the atom then measures the field parity while the injection of a coherent field with complex amplitude $\alpha$, before the atom-cavity interaction, performs the required displacement. This scheme should thus lead to the direct measurement of $W(\alpha)$. It does not involve an inversion algorithm and should be much less sensitive to experimental errors than the tomographic techniques.

The Wigner function corresponding to the $n=1$ Fock state for the center-of-mass motion of a trapped ion was measured at NIST [14], displaying clearly the negative region around the origin. Up to now, however, no similar measurement has ever been performed for electromagnetic fields. We report here a direct measurement of $W_1(0)$ and $W_0(0)$, the Wigner functions for the vacuum and for a single-photon field at the origin of phase space. The heart of our method is the absorption-free detection of a single-photon stored in a microwave cavity [15]. We first use a ''source'' atom to prepare a one-photon field [16]. A ''meter'' atom then measures the field parity and hence the Wigner function at the origin. A ''probe'' atom finally checks the cavity state at the end of the experiment. This third atom allows us to reject most errors due to cavity relaxation. The principle of this three-atom experiment is quite comparable to our recent preparation of a two-atom plus cavity entangled state (the cavity state being later read out by a third atom) [17]. The operating conditions of the present experiment are, however, different being optimized to improve the fidelity of the Wigner function determination.

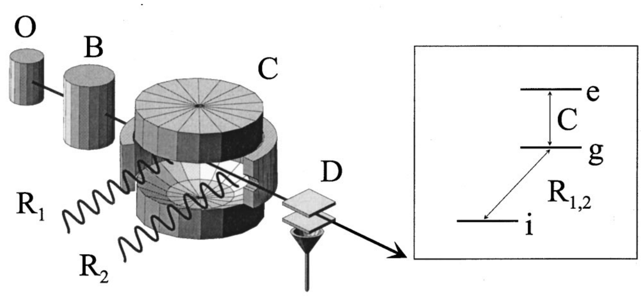


FIG. 2. Experimental setup for measuring the Wigner function of a zero or one-photon Fock state at the origin of phase space.

Let us recall the principle of the single-photon detection. Figure 2 presents the scheme of the experiment, with the relevant atomic states in the inset. The meter atom is initially prepared in level $g$. A first $\pi/2$ microwave pulse $R_1$, resonant with the $g\Rightarrow i$ transition, then prepares the superposition $(|g\rangle+|i\rangle)/\sqrt{2}$ and the meter interacts with the cavity $C$, resonant with the $g\Rightarrow e$ transition. When $C$ contains no photon, or when the meter is in state $i$, the meter's state is unaffected. When $C$ contains one photon, the meter in level $g$ undergoes a quantum Rabi nutation between $g$ and $e$. The atom-cavity interaction time is adjusted for a $2\pi$ rotation. The $|g,1\rangle$ state is thus transformed into $-|g,1\rangle$. The photon is left in $C$ and the atomic superposition becomes $(-|g\rangle+|i\rangle)/\sqrt{2}$. The meter states $|g\rangle$ and $|i\rangle$ are then mixed again by another $\pi/2$ pulse $R_2$, which realizes, together with $R_1$, a Ramsey interferometer [13]. The phase of $R_2$ is set so that it performs the transformations $(|g\rangle-|i\rangle)/\sqrt{2}\rightarrow|g\rangle$ and $(|g\rangle+|i\rangle)/\sqrt{2}\rightarrow|i\rangle$. The final meter state is thus $i$ ($g$) when the photon number is 0 (1).

Provided the cavity field state is in the $\{|0\rangle,|1\rangle\}$ subspace, the difference $P_i-P_g$ of the probabilities for detecting the atom in $i$ or $g$ is equal to the mean value of $\hat{P}$, i.e., half the Wigner function at origin:

$$W(0)=2(P_i-P_g)=2-4P_g. \tag{3}$$

Let us stress that this method only applies in the $\{|0\rangle,|1\rangle\}$ subspace. For larger fields, the $2\pi$ Rabi pulse condition is not enforced for all photon numbers and the atomic state is not simply related to $\hat{P}$. Thus, $W$ cannot be measured at points different from the origin, since the required field displacement would take it outside this subspace. Note, however, that a proposed tomographic method to reconstruct $\hat{\rho}$ [18] uses a displacement of the cavity field followed by a resonant atom-field interaction.

The experimental setup is similar to the one used in previous studies [8,15–17,19–21]. Levels $e$, $g$ and $i$ are the rubidium circular Rydberg states with principal quantum numbers 51, 50, and 49, respectively. Three velocity-selected atomic samples, corresponding to the source, meter and probe, are excited into the circular state in box $B$ from an atomic beam effusing from oven $O$. Each of them contains much less than one atom on the average, so that two-atom events for a single sample are negligible. The cavity $C$ is made of two spherical niobium mirrors in a Fabry-Perot configuration. It sustains a Gaussian mode (waist $w=6$ mm) resonant with the $e\Rightarrow g$ transition (51.1 GHz). The single-photon Rabi frequency at cavity center is $\Omega/2\pi=47$ kHz [19]. The atomic velocity is $V=503(\pm 2.5)$ $\mathrm{ms}^{-1}$, corresponding to an effective interaction time $t_i=\sqrt{\pi}w/V$ such that $\Omega t_i=2\pi$. The mirrors are surrounded by a cylindrical ring with 3-mm holes for atom access. This ring reduces photon losses and increases the photon decay time up to 1 ms. A small electric field is applied across the mirrors. It is used to Stark tune the atomic frequency in or out of resonance with $C$. Hence, the atom-cavity interaction time can thus be reduced to any fraction of $t_i$. The 54.3-GHz Ramsey pulses $R_1$ and $R_2$, resonant with the $g\Rightarrow i$ transition, are applied to the atoms inside the cavity structure. The final states of the atoms are analyzed in the field-ionization detector $D$.

We have performed the experiment with the $|0\rangle$ and $|1\rangle$ states. We first get rid of a 0.7 photon thermal field by a cooling procedure described in [15]. The residual photon number is less than 0.1. The initial field state is then produced by the source atom, which is prepared in $e$. It enters $C$ 75 $\mu$s after the end of the cooling sequence and undergoes a $\pi/2$ spontaneous emission pulse in it. Either it is detected in $e$ (50% of the cases), leaving $C$ empty, or in $g$, preparing a single photon. In this way, the Wigner functions for the $|0\rangle$ and $|1\rangle$ states are recorded simultaneously. The meter, performing the single-photon detection, and probe are sent 100 $\mu$s and 175 $\mu$s after the source. Note that, with this timing, chosen to optimize the parity operator determination fidelity, the source is detected in $D$ even before the probe enters $C$. The probe is prepared in $g$. It undergoes a $\pi$ Rabi rotation in the field of one photon. Hence, ideally, it is detected in $g$ if $C$ is empty and in $e$ if $C$ contains one photon. When the detected states of the source and probe agree (either $(e_1g_3)$ or $(g_1e_3)$, where the indices refer to the order of the atoms in the experiment), the field state has not been affected by cavity relaxation during the experimental sequence. In other words, the detection of the probe performs a ''post-selection'' of the field state experienced by the meter.

Figure 3 presents the observed atomic correlations. The hatched histograms give the probabilities for detecting the source and probe in the four possible channels *when no meter atom is sent*. One thousand two-atom coincidences have been recorded in about twenty minutes. Ideally, channels $g_1e_3$ and $e_1g_3$ should have identical 0.5 probabilities. The populations of the other channels ($g_1g_3$ and $e_1e_3$) mainly originate from cavity relaxation. Photon decay populates $g_1g_3$ from $g_1e_3$ (the probability of a decay between the source and the probe is 16%). The appearance of a thermal photon accounts in part for the population of the channel $e_1e_3$ (the corresponding probability is 13%). Other imperfections also contribute (thermal photon already present in $C$ before the source, two-atom events in one sample, detectors imperfections, etc.).

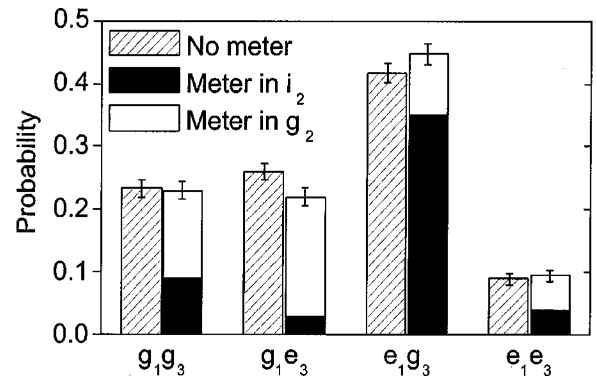


FIG. 3. Histograms giving the detection probabilities (about 1000 coincidences detected). Hatched bars: detection probabilities for the four possible output channels for the source and probe atom when no meter atom is sent through $C$. Black and white bars: detection probabilities for the source and probe conditioned to the state of the meter atom ($i_2$ and $g_2$ respectively). The error bars are statistical.

The results obtained when the meter is sent through $C$ are displayed by the black and white histograms. About 1000 three-atom coincidences have been recorded in two hours. These histograms give the probabilities for the eight possible outcomes. The two bars corresponding to a given configuration of the source and probe are stacked. Each white bar corresponds to a meter detected in $g_2$, while the black ones correspond to a detection in $i_2$.

We observe that, within statistical errors (depicted by the error bars in Fig. 3), the stacked black and white boxes have the same height as the corresponding hatched one. Hence, the meter does not alter the detection probabilities for the source and the probe and thus leaves the photon number unchanged [15]. For the $g_1e_3$ case (1 photon in $C$), the meter is detected mainly in $g_2$, in agreement with the settings of the Ramsey pulses (see above). Alternatively, it is detected most often in $i_2$ in the $e_1g_3$ case (empty cavity). The spurious channels mainly correspond to the absorption ($g_1g_3$) or to the creation ($e_1e_3$) of a photon. This occurs with equal probabilities before or after the meter has crossed $C$. In these cases, the meter is detected with almost equal probabilities in $g_2$ or $i_2$.

The relevant conditional probabilities are easily computed. The probability for detecting the meter in $g_2$ provided the cavity contains one photon ($g_1e_3$ case) is $P(g_2|g_1e_3)=0.83(4)$. In a similar way, the probability for detecting the meter in $g_2$ in the empty cavity case ($e_1g_3$ case) is found to be $P(g_2|e_1g_3)=0.22(3)$. These figures are consistent with the 20% error rate of the single-photon absorption-free detection reported in [15]. From Eq. (3), we infer the values of $W(0)$ in the zero and one photon cases:

$$W_0(0)=1.12(12), \tag{4}$$

$$W_1(0)=-1.32(16). \tag{5}$$

Even though the $\pm 2$ theoretical values are not reached due to the finite (60%) contrast of our Ramsey interferometer, the negativity of the Wigner function of a single-photon field at the origin of phase space is clearly demonstrated.

The efficiency of the error rejection provided by the probe atom can be estimated. If we discard the probe information, we get for the relevant conditional probabilities $P(g_2|g_1) = 0.71(4)$ for the one-photon case and $P(g_2|e_1) = 0.27(3)$ for the vacuum case, leading to the values $W_0(0) = 0.88(16)$ and $W_1(0) = -0.87(12)$. A very similar figure can be deduced from the data presented in [15]. The quality of the Wigner function determination is thus considerably improved by the probe atom detection.

In conclusion, we have shown that the absorption-free detection of a single photon in a cavity leads to the first measurement of a negative value for the Wigner function of an electromagnetic field. Further efforts are now being made in order to measure the Wigner function over the complete phase space. This requires a dispersive interaction [12] between the meter and $C$. The $\pi$ atomic wave function phase shift for a single photon in the dispersive regime could be reached in the present setup. In this case, the parity operator could be measured for fields containing up to a few photons, allowing for the complete determination of the Wigner function. Of particular interest would be the measurement of the Wigner function for a relaxing ''Schrödinger cat'' state [8]. The corresponding experiments are in progress.

Two of the authors (L.D. and L.G.L.) acknowledge the support of Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Coordenação de Aperfeiçoamento de Pessoal de Ensino Superior (CAPES), Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ), Fundação Universitária José Bonifácio (FUJB), and Programa de Apoio a Núcleos de Excelência (PRONEX). We acknowledge support of the European Community and of Japan Science and Technology Corporation (International Cooperative Research Project ''Quantum Entanglement'').

[1] F.T. Arecchi, Phys. Rev. Lett. **15**, 912 (1965); M.O. Scully and W.E. Lamb, Jr., Phys. Rev. **159**, 208 (1967); H. Haken, in *Handbuch der Physik*, edited by L. Genzel (Springer, Berlin, 1970), Vol. XXV/2C.

[2] H.J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977); Phys. Rev. A **18**, 201 (1978); R. Short and L. Mandel, Phys. Rev. Lett. **51**, 384 (1983).

[3] R.E. Slusher *et al.*, Phys. Rev. Lett. **55**, 2409 (1985); J. Opt. Soc. Am. B **4** 1449 (1987), special issue on squeezed states of the electromagnetic field, edited by H.J. Kimble and D.F. Walls; J. Mod. Opt. **34** 707 (1987), special issue on squeezed light, edited by R. Loudon and P.L. Knight; Appl. Phys. B: Photophys. Laser. Chem. **55** 189 (1992), special issue on quantum noise reduction in optical systems—experiments, edited by E. Giacobino and C. Febre.

[4] E. Wigner, Phys. Rev. **40**, 749 (1932); for a review, see M. Hillery *et al.*, Phys. Rep. **106**, 121 (1984).

[5] J.E. Moyal, Proc. Cambridge Philos. Soc. **45**, 99 (1949).

[6] K.E. Cahill and R.J. Glauber, Phys. Rev. **177**, 1857 (1969); **177**, 1882 (1969).

[7] V. Buzek and P. L. Knight, *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1995), Vol. XXXIV, p. 1, and references therein.

[8] M. Brune *et al.*, Phys. Rev. Lett. **77**, 4887 (1996).

[9] J. Bertrand and P. Bertrand, Found. Phys. **17**, 397 (1987); K. Vogel and H. Risken, Phys. Rev. A **40**, 2847 (1989); see also J. Mod. Opt. **44** (11) (1997), special issue on quantum state preparation and measurement, edited by W.P. Schleich and M.G. Raymer.

[10] D.T. Smithey *et al.*, Phys. Rev. Lett. **70**, 1244 (1993); G. Breitenbach *et al.*, J. Opt. Soc. Am. B **12**, 2304 (1995); S. Schiller *et al.*, Phys. Rev. Lett. **77**, 2933 (1996).

[11] K. Banaszek and K. Wódkiewicz, Phys. Rev. Lett. **76**, 4344 (1996); S. Wallentowitz and W. Vogel, Phys. Rev. A **53**, 4528 (1996); K. Banaszek *et al.*, *ibid.* **60**, 674 (1999).

[12] L.G. Lutterbach and L. Davidovich, Phys. Rev. Lett. **78**, 2547 (1997); L.G. Lutterbach and L. Davidovich, Opt. Express **3**, 147 (1998).

[13] N. F. Ramsey, *Molecular Beams* (Oxford University Press, New York, 1985).

[14] D. Leibfried *et al.*, Phys. Rev. Lett. **77**, 4281 (1996).

[15] G. Nogues *et al.*, Nature (London) **400**, 239 (1999).

[16] X. Maitre *et al.*, Phys. Rev. Lett. **79**, 769 (1997).

[17] A. Rauschenbeutel *et al.*, Science **288**, 2024 (2000).

[18] M.S. Kim, G. Antesberger, C.T. Bodendorf, and H. Walther, Phys. Rev. A **58**, R65 (1998).

[19] M. Brune *et al.*, Phys. Rev. Lett. **76**, 1800 (1996).

[20] E. Hagley *et al.*, Phys. Rev. Lett. **79**, 1 (1997).

[21] A. Rauschenbeutel *et al.*, Phys. Rev. Lett. **83**, 5166 (1999).