Laser Physics, Vol. 10, No. 1, 2000, pp. 42–47.

MODERN TRENDS IN LASER PHYSICS

Quantum Theory of Microlasers in the Close-to-Threshold Regime

P. Domokos*[1], J. Hare*, V. Lefèvre*, J. M. Raimond*, S. Haroche*, I. Protsenko, and L. Davidovich*****

** Laboratoire Kastler Brossel [2], Département de Physique de l'Ecole Normale Supérieure, 24 rue Lhomond, Paris, 75231 France*

*** Lebedev Physics Institute, Leninskii pr. 53, Moscow, 117924 Russia*

**** Instituto do Fisica, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ, 21945-970 Brasil*

e-mail: domokos@power.szfki.kfki.hu

Received September 11, 1999

Abstract—We present a new analytical method to solve the basic Heisenberg–Langevin equations of the standard quantum model of a single-mode laser. The method is valid for describing microlasers operating in the strong coupling regime (cavity QED lasers). These lasers can exhibit fascinating properties: for example, the very notion of threshold may break down. Owing to the analytic solutions of the new model, the laser operation in the close-to-threshold regime can be studied, which leads us to a natural characterization of the threshold's behavior.

In numerous laboratories, experiments aiming to construct microlasers are in course. The heart of these devices is a miniature cavity whose geometry is tailored so that the emission into other than the laser mode is reduced. The ultimate goal is the complete elimination of the radiation loss and, hence, the realization of a thresholdless laser [1].

Different systems have been conceived to observe genuine microlaser effects. Thresholdless lasers were reported, for example, in multilayer semiconductor microcavities and in microdisks [2]. A very promising candidate consists in using the whispering gallery modes of a high-Q, fused-silica microsphere. Light in such modes is trapped in the equatorial plane near to the surface by total internal reflection. The mode is confined in a small volume (300 μm^3) without increasing the diffraction loss (quality Q factor up to 10^9 can be achieved, limited by the absorption in the material). By means of a high index prism, the total reflection at the sphere's internal surface can be frustrated in order to couple photons into (or out of) the mode. The active medium is composed of Nd^{3+} ions embedded in silica by doping. Laser operation, exhibiting very low threshold, has recently been demonstrated in this system [3].

The microsphere setup realizes a cavity QED laser [4]. The essential difference with respect to usual lasers resides in the coupling between the active medium and the laser mode. Due to the small mode volume in a microcavity, the interaction can be in the strong coupling regime. The nature of spontaneous emission changes significantly and, hence, the quantum statistical properties of the emitted light are profoundly influenced. The description of such a system demands cavity QED techniques.

Motivated by the microsphere experiments, we have developed a quantum theory that can account for the measurable characteristics of microlasers even in the close-to-threshold regime. In this paper, we present the analytical model in full generality, without concentrating on a specific configuration of the operating parameters. In Section 1, the basic equations of the standard quantum theory based on first principles will be presented. The new approximate quantum model will be established in Section 2. Finally, Section 3 is devoted to the solution; especially, the mean intensity, the linewidth, and the intensity noise will be calculated.

1. THE BASIC HEISENBERG–LANGEVIN EQUATIONS

We briefly summarize the basic equations of the quantum-mechanical model for a single-mode laser. The active medium is composed of two-level atoms (upper level a, lower level b), schematically represented in Fig. 1. For the sake of simplicity, no inhomogeneous effects will be considered (the generalization can be carried out following the work [5]). That is, all the atoms are uniformly coupled to a single resonant cavity mode with a coupling constant g. The homogeneous linewidth of the gain medium is Γ, which can be in arbitrary relation with the cavity mode damping rate κ. We also assume that the number of accessible atoms is sufficiently large compared to the number of the actually active atoms, so that the pumping process can be considered Poissonian. The pumping can then be

[1] Permanent address: Research Institute for Solid State Physics and Optics, P.O. Box 49, H-1525, Hungary.

[2] Laboratoire de l'Université Pierre et Marie Curie et de l'ENS, associe au CNRS (URA18).

described by a single parameter R, the rate of excitation into the state a. In interaction picture, the following Heisenberg–Langevin equations can be derived, as shown in [6]:

$$\dot{a}(t) = gM(t) - \frac{\kappa}{2}a(t) + F_\kappa(t), \quad (1a)$$

$$\dot{M}(t) = g[N_a(t) - N_b(t)]a(t) - \Gamma M(t) + F_M(t), \quad (1b)$$

$$\dot{N}_a(t) = R - g[a^\dagger(t)M(t) + M^\dagger(t)a(t)] - \Gamma_a N_a(t) + F_a(t), \quad (1c)$$

$$\dot{N}_b(t) = g[a^\dagger(t)M(t) + M^\dagger(t)a(t)] - \Gamma_b N_b(t) + F_b(t), \quad (1d)$$

where a, $a^\dagger$ are the boson operators of the field mode; M is the collective atomic polarization; and N_a and N_b are the populations in levels a and b, respectively. The noise features are incorporated in the F reservoir operators. They obey the usual Langevin correlations

$$\langle F_i(t)\rangle = 0, \quad \langle F_i(t)F_j(t')\rangle = 2D_{ij}\delta(t-t'), \quad (2)$$

where the nonvanishing diffusion coefficients at zero temperature are

$$2D_{\kappa\kappa^\dagger} = \kappa,$$
$$2D_{M^\dagger M} = (2\Gamma - \Gamma_a)\langle N_a(t)\rangle + R,$$
$$2D_{MM^\dagger} = (2\Gamma - \Gamma_b)\langle N_b(t)\rangle,$$
$$2D_{aa} = \Gamma_a\langle N_a(t)\rangle + R, \quad (3)$$
$$2D_{bb} = \Gamma_b\langle N_b(t)\rangle,$$
$$2D_{bM} = \Gamma_b\langle M(t)\rangle,$$
$$2D_{Ma} = \Gamma_a\langle M(t)\rangle.$$

There is no general solution for these equations. One may resort to various approximations [7–9] for generating solutions in special cases. However, none of the usual procedures can be applied to a cavity QED laser system. The aim of this paper consists in presenting a new method that enables us to find approximative solutions for the operator variables a and M, valid in the close-to-threshold regime of a cavity QED laser.

Beforehand, let us recall the Lamb semiclassical model, which can serve as a source of useful scaling parameters for our solutions. It can be recovered from (1) by considering all the operators as mere c-numbers and suppressing the noise terms. The steady-state solution can be easily obtained without any further approximation. The oscillation threshold is given by

$$R_{\rm th} = \frac{\kappa\Gamma\Gamma_a}{2g^2}. \quad (4)$$

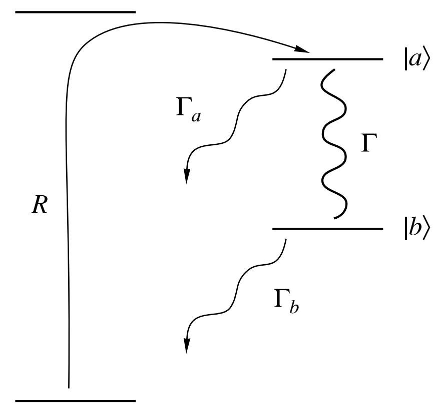

Fig. 1. Relevant level scheme.

If $R < R_{\rm th}$, the semiclassical steady-state intensity is equal to zero and the steady-state population inversion is given by R/Γ_a. For $R \geq R_{\rm th}$, the semiclassical steady-state mean photon number is given by

$$I_0 = I_{\rm sat}(R/R_{\rm th} - 1), \quad (5)$$

where the "saturation intensity," $I_{\rm sat}$, is defined by

$$I_{\rm sat} = \frac{\Gamma\Gamma_a\Gamma_b}{2g^2(\Gamma_a + \Gamma_b)} = \frac{R_{\rm th}}{\kappa}\frac{1}{1 + \Gamma_a/\Gamma_b}. \quad (6)$$

Above threshold, the population inversion is independent of the pumping rate (this is the "population clamping" effect characteristic of homogeneously broadened lasers) and given by

$$\Delta_0 = \frac{\kappa\Gamma}{2g^2}. \quad (7)$$

The Lamb model is expected to coincide with the solution of our model when the laser has a well-defined threshold and operates far above it.

2. APPROXIMATE QUANTUM MODEL

The basic assumption of our analytical treatment is that the population inversion is large enough so that its fluctuations can be neglected. This hypothesis is reasonable for many laser systems, especially for microlasers where the number of atoms taking part in the interaction is typically much larger than the generated photon number.

The crucial step in applying this assumption is the replacement of the population inversion operator by its mean value in (1b). Since we do not consider the fluctuations of the atomic populations, we may replace (1c) and (1d) by their quantum-mechanical means. The new set of equations is then

$$\dot{a}(t) = gM(t) - \frac{\kappa}{2}a(t) + F_\kappa(t), \quad (8a)$$

$$\dot{M}(t) = g\langle N_a(t) - N_b(t)\rangle a(t) - \Gamma M(t) + F_M(t), \quad (8b)$$

$$\langle \dot{N}_a(t)\rangle = R - g\langle a^\dagger(t)M(t) + M^\dagger(t)a(t)\rangle - \Gamma_a\langle N_a(t)\rangle, \quad (8c)$$

$$\langle \dot{N}_b(t)\rangle = g\langle a^\dagger(t)M(t) + M^\dagger(t)a(t)\rangle - \Gamma_b\langle N_b(t)\rangle. \quad (8d)$$

For the systems under consideration, the intensities involved may be very low, down to a few photons. Hence, we are not allowed to linearize these equations around the steady-state values. On the other hand, we will be interested in the behavior of the steady-state and (8) should therefore be considered after the transients die out and all average quantities become constants. The essential part of the Heisenberg–Langevin equations left to be solved is

$$\dot{a}(t) = gM(t) - \frac{\kappa}{2}a(t) + F_\kappa(t), \quad (9a)$$

$$\dot{M}(t) = g\Delta a(t) - \Gamma M(t) + F_M(t), \quad (9b)$$

which comprises coupled linear differential equations containing the population inversion as a c-number constant parameter Δ. Note that the inherent nonlinearity of the original problem is not masked in this approach. Although the above set of equations is formally linear for the variables a and M, the value of the parameter Δ depends on the field amplitude a and on the polarization M. Therefore, the solution must be generated on a self-consistent manner. First, we find a and M as a function of Δ from (9). Then, these solutions are inserted into equations (8c) and (8d) in order to determine the actual value of the inversion Δ. This latter step can be simplified, since, in the stationary operation, the population inversion is connected to the intensity $I = \langle a^\dagger a\rangle$ by

$$\frac{\Delta - \Delta_0}{\Delta_0} = \frac{I_0 - I}{I_{\text{sat}}}. \quad (10)$$

This balance equation, which can be deduced from (8a), (8c), and (8d) in the present model, must hold in all single-mode two-level laser models at zero temperature. For example, the semiclassical solution corresponds to the special case when both sides of the (10) are zero.

In the last step, to make the model complete, it is necessary to redefine the diffusion coefficients. Having replaced the population inversion operator by a c-number constant, we might have changed the algebraic properties of the atomic system. Indeed, the commutation rule for the polarization, which is written as

$$[M^\dagger(t), M(t)] = N_a - N_b, \quad (11)$$

is now approximated by

$$[M^\dagger(t), M(t)] \approx \Delta = \text{constant}. \quad (12)$$

Therefore, our original physical hypothesis implies that the atomic polarization operator behaves like a bosonic operator

$$M(t) = \begin{cases} \sqrt{|\Delta|}\,b^\dagger(t), & \text{if} \quad \Delta \ge 0, \\ \sqrt{|\Delta|}\,b(t), & \text{if} \quad \Delta < 0, \end{cases} \quad (13)$$

where $[b(t), b^\dagger(t)] = 1$. Consequently, the associated Langevin noise operators should obey the commutation relation

$$\langle [F_M^\dagger(t), F_M(t')]\rangle = 2\Gamma\Delta\delta(t - t'), \quad (14)$$

just as the field noise F_κ does. In conclusion, for the sake of the algebraic consistency, it is necessary to redefine the diffusion coefficients so that

$$2D_{M^\dagger M} - 2D_{MM^\dagger} = 2\Gamma\Delta. \quad (15)$$

Later we will prove that this redefinition is also sufficient for maintaining the relevant algebraic rules. A necessary condition for the validity of the model can be obtained by comparing (15) to the original definition

$$2D_{M^\dagger M} - 2D_{MM^\dagger} = 2\Gamma\Delta + R - \Gamma_a\langle N_a\rangle + \Gamma_b\langle N_b\rangle. \quad (16)$$

From the fact that the difference between (15) and (16) has to be small, we get

$$\Delta \gg (\kappa/\Gamma)I, \quad (17)$$

which is the mathematical formulation of the basic physical hypothesis, namely, the population inversion must be considerably larger than the number of photons. We may therefore expect that this model works in the close-to-threshold regime.

Since only the difference between the diffusion coefficients $2D_{M^\dagger M}$ and $2D_{MM^\dagger}$ is tied up by the condition (15), there is some freedom to define the values of $2D_{M^\dagger M}$ and $2D_{MM^\dagger}$. We thus set

$$2D_{M^\dagger M} = 2\Gamma\Delta, \quad 2D_{MM^\dagger} = 0. \quad (18)$$

This choice, slightly different from that used in [11], will lead to the simplest form of the results in later calculations. The usual diffusion coefficients associated with the field amplitude will be retained:

$$2D_{\kappa\kappa^\dagger} = \kappa, \quad 2D_{\kappa^\dagger\kappa} = 0. \quad (19)$$

Furthermore, since both the amplitude and the polarization now behave as bosonic operators, the associated Langevin noise can be supposed Gaussian. As a consequence, the knowledge of the second-order correlations is enough to calculate higher order moments.

3. SOLUTION

Equations (9) and (10) and the definition of the diffusion coefficients in (18) and (19) set up a complete model to calculate the field amplitude a, the atomic polarization M, and the mean population inversion Δ for arbitrary pumping rate R. Before integrating the first-order differential equations (9), it is worth inspecting the eigenvalues of the homogeneous part of the linear system:

$$\lambda_{\pm} = -\frac{1}{2}(\kappa/2 + \Gamma) \pm \frac{1}{2}\sqrt{(\kappa/2 + \Gamma)^2 + 4g^2(\Delta - \Delta_0)}. \quad (20)$$

The stability of the solutions requires that both of its root should be negative. That is, we get the condition

$$\Delta \le \Delta_0. \quad (21)$$

The population inversion predicted by the model must be inferior to its semiclassical value. This corroborates our physical intuition: since our approach takes into account the spontaneous emission of photons into the laser mode, less population inversion is needed to balance the photon loss.

The explicit solution for the field amplitude and for the polarization can be expressed in terms of the Langevin noise operators as follows:

$$a(t) = \frac{1}{\mathcal{N}^2}\int_0^\infty d\tau[\chi_{\kappa\kappa}(\tau)F_\kappa(t-\tau) + \chi_{\kappa M}(\tau)F_M(t-\tau)], \quad (22a)$$

$$M(t) = \frac{1}{\mathcal{N}^2}\int_0^\infty d\tau[\chi_{M\kappa}(\tau)F_\kappa(t-\tau) + \chi_{MM}(\tau)F_M(t-\tau)], \quad (22b)$$

where

$$\chi_{\kappa\kappa}(\tau) = -(\kappa/2 + \lambda_-)(\Gamma + \lambda_+)e^{\lambda_+\tau} + g^2\Delta e^{\lambda_-\tau}, \quad (23a)$$

$$\chi_{\kappa M}(\tau) = (\Gamma + \lambda_+)g(e^{\lambda_+\tau} - e^{\lambda_-\tau}), \quad (23b)$$

$$\chi_{M\kappa}(\tau) = -(\kappa/2 + \lambda_-)g\Delta(e^{\lambda_+\tau} - e^{\lambda_-\tau}), \quad (23c)$$

$$\chi_{MM}(\tau) = g^2\Delta e^{\lambda_+\tau} - (\kappa/2 + \lambda_-)(\Gamma + \lambda_+)e^{\lambda_-\tau}, \quad (23d)$$

and

$$\mathcal{N}^2 = g^2\Delta - (\kappa/2 + \lambda_-)(\Gamma + \lambda_+). \quad (24)$$

Any measurable quantity of interest can be calculated from the solution (22). Since only the mean values of second-order products of the Langevin noise operators do not vanish, one needs integrals of the form

$$\mathcal{I}_{ij,kl} = \int_0^\infty \chi_{ij}(\tau)\chi_{kl}(\tau)d\tau, \quad (25)$$

where the ij, kl index is an arbitrary combination of κ and M. The independent integrals are

$$\mathcal{I}_{\kappa\kappa,\kappa\kappa} = \frac{\Gamma^2}{D} + \frac{1}{2}\frac{1}{\kappa/2 + \Gamma}, \quad (26a)$$

$$\mathcal{I}_{MM,MM} = \frac{(\kappa/2)^2}{D} + \frac{1}{2}\frac{1}{\kappa/2 + \Gamma}, \quad (26b)$$

$$\mathcal{I}_{\kappa M,\kappa M} = \frac{g^2}{D}, \quad (26c)$$

$$\mathcal{I}_{\kappa\kappa,MM} = \frac{\Gamma\kappa/2}{D} + \frac{1}{2}\frac{1}{\kappa/2 + \Gamma}, \quad (26d)$$

$$\mathcal{I}_{\kappa\kappa,\kappa M} = \frac{g\Gamma}{D}, \quad (26e)$$

$$\mathcal{I}_{MM,\kappa M} = \frac{g\kappa/2}{D}, \quad (26f)$$

where

$$D = 2g^2(\Delta_0 - \Delta)(\kappa/2 + \Gamma). \quad (27)$$

The other integrals can easily be derived from the above set by noticing that $\mathcal{I}_{M\kappa,ij} = \Delta\mathcal{I}_{\kappa M,ij}$ and $\mathcal{I}_{ij,kl} = \mathcal{I}_{kl,ij}$.

We can verify that the solutions (22) are consistent with the relevant algebraic rules. The model is restricted to represent only the field amplitude a and the polarization M by operators. The populations are included just by their means; and hence, it makes no sense to demand such relations as $M^\dagger M = N_a$, $MM^\dagger = N_b$. Nevertheless, the requirement of algebraic consistency, which has been imposed by the proper choice of the diffusion coefficients, should manifest itself by the validity of the relation $\langle M^\dagger M - MM^\dagger\rangle = \Delta$. One can check that

$$\langle M^\dagger M - MM^\dagger\rangle = (2D_{M^\dagger M} - 2D_{MM^\dagger})\mathcal{I}_{MM,MM} - 2D_{\kappa\kappa*}\mathcal{I}_{M\kappa,M\kappa} \quad (28)$$

provides the expected result Δ as far as $(2D_{M^\dagger M} - 2D_{MM^\dagger}) = 2\Gamma\Delta$. So the previous definition proves to be necessary and sufficient to ensure the usual operator algebra.

3.1. Intensity and Threshold Behavior

Let us calculate the intensity of the laser mode:

$$I = \langle a^\dagger a\rangle = \frac{1}{\mathcal{N}^4}\int_0^\infty d\tau\int_0^\infty d\tau'\chi_{\kappa M}(\tau)\chi_{\kappa M}(\tau')\langle F_M^\dagger(t-\tau)F_M(t-\tau')\rangle = 2D_{M^\dagger M}\mathcal{I}_{\kappa M,\kappa M} = \frac{\Gamma\Delta}{(\kappa/2 + \Gamma)(\Delta_0 - \Delta)}. \quad (29)$$

This expression, together with the balance equation (10), yields a closed set of equations for the population inversion and for the intensity. It leads to a second-

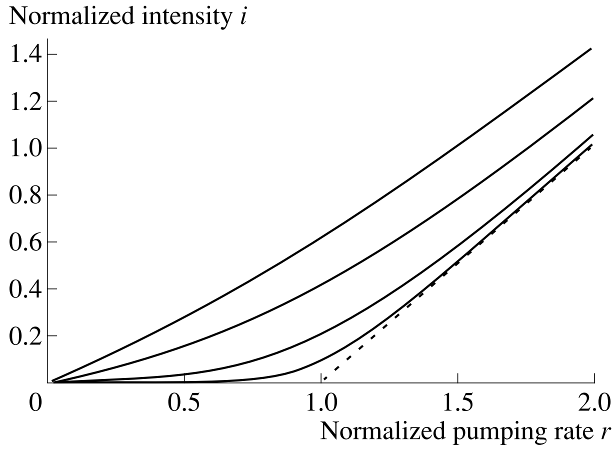

Fig. 2. Normalized intensity $i = I/I_{sat}$ versus normalized pumping rate $r = R/R_{th}$. The parameter c is chosen to be 0.008, 0.05, 0.3, and 1. Dashed line represents the semiclassical solution with a threshold at $r = 1$. For the smallest c value ($c = 0.008$), the semiclassical threshold is still apparent. When c increases, the "threshold" comes closer to zero pumping, until it finally disappears for $c = 1$.

order algebraic equation, of which the solution for the normalized intensity $i = I/I_{sat}$ is

$$i = \frac{1}{2}(r-1-c) + \frac{1}{2}\sqrt{(r-1-c)^2 + 4cr}, \tag{30}$$

where the normalized pumping rate is $r = R/R_{th}$. The parameter c, which is defined by

$$c = \frac{1}{I_{sat}(1+\kappa/2\Gamma)(1+\Gamma_a/\Gamma_b)} = \frac{2g^2}{\Gamma_a(\Gamma_{ab}+\kappa/2)} = \frac{W}{\Gamma_a}, \tag{31}$$

has a very transparent physical meaning. It expresses the ratio of two dissipation rates: W is the rate of spontaneous emission into the laser mode and Γ_a is the relaxation rate, associated with all the *other* dissipation channels (collisions, spontaneous emission into lateral modes, etc.), of the population a.

The intensity is plotted as a function of the pumping rate in Fig. 2. According to the result (30), the scaled quantities $i = I/I_{sat}$ and $r = R/R_{th}$ are displayed. Curves corresponding to different values of the only relevant system-specific parameter c are represented. A reasonable definition for the threshold is the position of the maximum curvature, which can be found at $r = 1 - c$. The value of the curvature at this point can characterize the threshold quantitatively. For $c = 0.008$, a well-defined threshold occurs close to the semiclassical threshold $r = 1$. On enhancing c, less and less accentuated thresholds can be observed. Finally, the intensity versus pumping rate curve does not have a maximum curvature point; and, consequently, it makes no sense to use the notion of threshold any more. This happens in the parameter range

$$c \geq 1, \tag{32}$$

which we can consider as the condition of thresholdless laser.

Although the validity of the model was predicted to be the close-to-threshold regime, it can be shown that the model gives reasonable result for the intensity in the strong pumping regime as well. The asymptotic behavior ($r \longrightarrow \infty$) renders precisely the semiclassical result

$$i - i_0 \approx \frac{c}{i_0 + c} \longrightarrow 0. \tag{33}$$

3.2. Dynamical Behavior and Linewidth

Since it is assumed here that there is no detuning between the cavity mode and the atomic medium, the laser frequency coincides with them as well. The shape of the spectrum can be calculated by the Fourier transform of

$$\langle a^\dagger(t+T)a(t)\rangle = \langle a^\dagger(t)a(t)\rangle\left\{\frac{\lambda_+}{\lambda_+-\lambda_-}e^{\lambda_- T} - \frac{\lambda_-}{\lambda_+-\lambda_-}e^{\lambda_+ T}\right\}. \tag{34}$$

Our model provides, then, two degenerate lines in the spectrum with linewidths $|\lambda_\pm|$. The values of $|\lambda_\pm|$, which are the relaxation constants associated with the quasi-linear system (9), have already been presented at the beginning of this section. The corresponding weights are proportional to $|\lambda_\mp|/(\lambda_+ - \lambda_-)$; that is, the slower relaxation dominates. For zero pumping, $|\lambda_+|$ and $|\lambda_-|$ become equal to Γ and $\kappa/2$, respectively, reflecting the independence of the atomic and the field relaxations in this limit. For strong enough pumping, the lower relaxation rate symmetrically assembles the atomic and the field contribution, $|\lambda_-| = \kappa/2 + \Gamma$, while the dominant line has the width

$$|\lambda_+| \longrightarrow \frac{\Gamma\kappa/2}{\kappa/2+\Gamma}\frac{c}{i_0} = \frac{\Gamma^2\kappa/2}{(\kappa/2+\Gamma)^2}\frac{\Gamma_b}{\Gamma_a+\Gamma_b}\frac{1}{I_0}. \tag{35}$$

This latter obeys the well-known Schalow–Townes law, which again shows that, in the strong pumping regime, the predictions of our model are in complete agreement with the semiclassical theory. The variation of the linewidth $|\lambda_+|$ in a larger range of the pumping rate is displayed in Fig. 3.

3.3. Quantum Statistics of the Field

The analytical solutions (22) allow us to calculate higher order quantities. In particular, the intensity noise

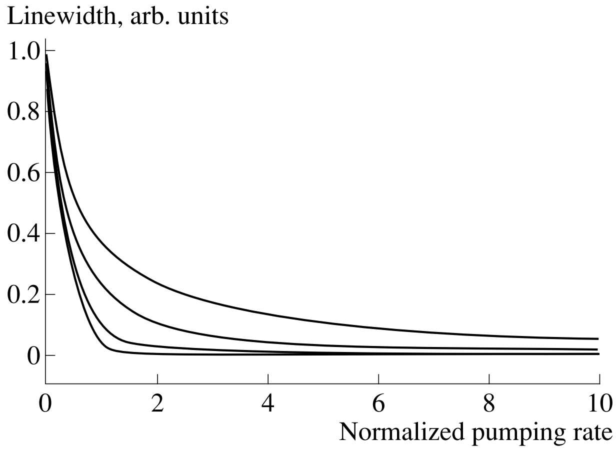

Fig. 3. The linewidth versus normalized pumping rate $r = R/R_{th}$. The parameters c are chosen to be the same as for the intensity plot, c = 0.008, 0.05, 0.3, and 1. The decay constants are set as $\kappa/2 = \Gamma = 1$.

of the laser field is of interest. A tedious calculation leads to the result

$$\langle a^\dagger a a^\dagger a\rangle = 2D_{M^\dagger M}(2D_{M^\dagger M} + 2D_{MM^\dagger})\mathcal{I}^2_{\kappa M,\kappa M} + 2D_{M^\dagger M}2D_{\kappa\kappa*}\mathcal{I}_{\kappa M,\kappa M}\mathcal{I}_{\kappa\kappa,\kappa\kappa} = 2\langle a^\dagger a\rangle^2 + \langle a^\dagger a\rangle. \quad (36)$$

The relation characteristic of a thermal field has been obtained. This is not surprising, since the solutions (22) are composed of the linear combinations of independent Gaussian noise operators. It is well known that the laser field well below threshold is indeed in a thermal state. The approximation of the field by a thermal field, a consequence of the original assumption of the present model, may also be good close to the threshold in a microlaser system. However, this approximation evidently fails for an intense semiclassical laser field, which approaches a coherent state rather than a thermal state. Note, however, that the mean intensity, as well as the linewidth (second-order quantities), are correctly found in the model for arbitrary pumping rate.

CONCLUSION

A new approximate quantum model has been established based on the Heisenberg–Langevin equations of a homogeneous laser. The model provides analytical solutions for any configuration of the linewidths Γ, κ, and coupling constant g. In the close-to-threshold regime, the results prove to be reliable in all orders; and they are also correct in the full range of the pumping rate as far as second-order quantities are considered. Starting from the intensity versus pumping rate curve, a measurable signal, we presented a natural characterization of the threshold's behavior and a reasonable definition of the thresholdless laser.

ACKNOWLEDGMENTS

One of us (P. D.) acknowledges the support by the National Scientific Fund of Hungary (OTKA) under Contract no. T023777. L. D. acknowledges the financial support of CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico), PRONEX (Programa de Apoio a Núcleos de Excelência), FAPERJ (Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro), and FUJB (Fundação Universitária José Bonifácio).

REFERENCES

1. De Martini, F. and Jacobovitz, G.R., 1988, *Phys. Rev. Lett.*, **60**, 1711.
2. Yamamoto, Y. and Slusher, R.E., 1993, *Phys. Today*, **46**, 66, and references therein.
3. Sandoghdar, V., Treussart, F., Hare, J., *et al.*, 1996, *Phys. Rev. A*, **54**, R1777.
4. Rice, P. and Carmichael, H., 1994, *Phys. Rev. A*, **50**, 4318.
5. Khoury, A.Z., Kolobov, M.I., and Davidovich, L., 1996, *Phys. Rev. A*, **53**, 1120.
6. Davidovich, L., 1997, *Rev. Mod. Phys.*, **68**, 127.
7. Benkert, C., Scully, M.O., Bergou, J., *et al.*, 1990, *Phys. Rev. A*, **41**, 2756.
8. Kolobov, M.I., Davidovich, L., Giacobino, E., and Fabre, C., 1993, *Phys. Rev. A*, **47**, 1431.
9. Prasad, S., 1992, *Phys. Rev. A*, **46**, 1540.
10. Lamb, W.E., Jr., 1964, *Phys. Rev.*, **134**, 1429; Louisell, W.H., 1990, *Quantum Statistical Properties of Radiation*, (John Wiley & Sons).
11. Protsenko, I., Domokos, P., Lefèvre-Seguin, V., *et al.*, 1999, *Phys. Rev. A*, **59**, 1667.