nature Vol 440|20 April 2006|doi:10.1038/nature04627

LETTERS

Experimental determination of entanglement with a single measurement

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Nearly all protocols requiring shared quantum information[1]—such as quantum teleportation[2] or key distribution[3]—rely on entanglement between distant parties. However, entanglement is difficult to characterize experimentally. All existing techniques for doing so, including entanglement witnesses[4,11,12] or Bell inequalities[5], disclose the entanglement of some quantum states but fail for other states; therefore, they cannot provide satisfactory results in general. Such methods are fundamentally different from entanglement measures that, by definition, quantify the amount of entanglement in any state. However, these measures suffer from the severe disadvantage that they typically are not directly accessible in laboratory experiments. Here we report a linear optics experiment in which we directly observe a pure-state entanglement measure, namely concurrence[6]. Our measurement set-up includes two copies of a quantum state: these 'twin' states are prepared in the polarization and momentum degrees of freedom of two photons, and concurrence is measured with a single, local measurement on just one of the photons.

The functional relation between an entanglement measure and the state to be characterized is typically complicated and nonlinear. Even worse, many measures rely on operations that cannot be implemented in laboratory experiments for fundamental reasons: they do not preserve the positivity of general quantum states. That is, these operations would spoil the statistical interpretation of quantum mechanics and thus cannot be realized in any physical system. In particular, this holds for the most commonly used measures, namely concurrence[7] and negativity[8]. Thus, it is necessary to measure a complete set of observables, reconstruct the system state, and eventually evaluate them. This—although possible and successfully implemented[9,10] for relatively small systems—becomes virtually impossible with larger systems because the number of observables to be measured grows exponentially with the number of entangled subsystems. On the other hand, one might expect that a property like entanglement, which is believed to constitute one of the most remarkable differences between classical and quantum systems, should have signatures that can be observed directly. And indeed, there is a proposal[13] to directly observe concurrence by replacing the underlying unphysical operation by some physical approximation. This, however, requires a rather involved experimental set-up, and has not been realized yet.

Here, we report the direct experimental observation of an entanglement measure, namely concurrence[7]

$$C = |\langle \Psi^{*}|\sigma_y \otimes \sigma_y|\Psi\rangle|, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad (1)$$

originally defined in terms of the second Pauli matrix σ_y and the complex conjugate $\langle \Psi^{*}|$ of the original state $\langle \Psi|$. The problems that arise owing to the unphysical operation of complex conjugation in equation (1) are overcome here with a generalization of concurrence[14] to systems of arbitrary finite dimensions, which, if restricted to qubits, is equivalent to the original concurrence. In this case the nonlinear dependence on the system state—that constitutes a fundamental property of any entanglement measure—is taken into account by considering a twofold copy of the state in question. Indeed, it has been shown that any mth degree polynomial function of a density matrix ρ can be measured on an m-fold copy of ρ (ref. 15). More precisely, the concurrence C of an arbitrary state $|\Psi\rangle$ can be defined as $C = 2\sqrt{P_A}$, where $P_A = \langle \Psi|\otimes\langle \Psi|A|\Psi\rangle\otimes|\Psi\rangle$ is the probability of observing the two copies of the first subsystem in an antisymmetric state, that is, a state that acquires a phase shift of π upon exchange of the constituents, and A is the corresponding measurement operator. In particular, no measurement needs to be performed on either copy of the second subsystem—though, in general, detections on the second subsystem can be used to trigger detectors for measurements on the first subsystem.

In our specific set-up, shown in Fig. 1, we created two copies of a bipartite quantum state using a photon pair obtained by spontaneous parametric down-conversion, where the polarization and momentum degrees of freedom each store one copy of the state $|\Psi\rangle$,

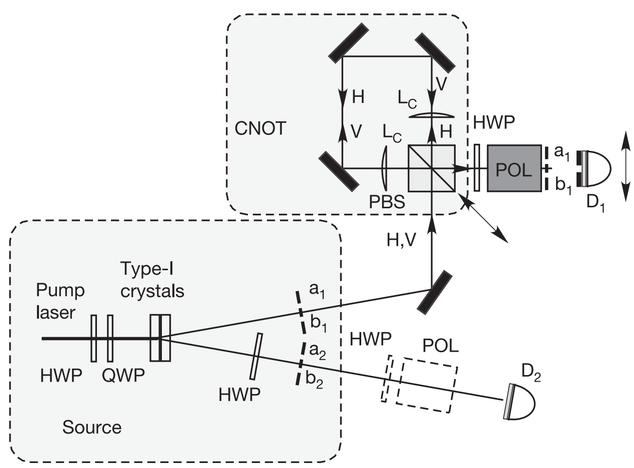

Figure 1 | Experimental set-up for the measurement of entanglement using two copies of the quantum state. Photon pairs that bear entanglement in two different degrees of freedom were created by pumping two type-I $LiLO_3$ crystals with a 200 mW HeCd continuous-wave laser (442 nm). Double-square apertures (1 mm × 1 mm squares, 2 mm centre to centre separation) placed 1 m from the crystal face are used to select distinct spatial modes a and b. Detectors D_1 and D_2 use 1.4 mm circular and 1 × 5 mm rectangular detection apertures, respectively. Both were equipped with interference filters (full-width at half-maximum, 10 nm). HWP, half-wave plate; QWP, quarter-wave plate; PBS, polarizing beam splitter; L_C, cylindrical lens; POL, polarization filter. CNOT, controlled-not. H and V indicate horizontal and vertical polarization.

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respectively[16,17]. That is, both copies required for our measurement are carried by the same photon, which significantly facilitates the set-up, as a measurement on only a single photon will be necessary. In fact, any required measurement can be realized efficiently with linear optics only[18,19].

Entangled polarization states were prepared by pumping two perpendicular nonlinear crystals with a continuous-wave laser beam[20]. The additional help of half- and quarter-wave plates placed in the path of both the pump beam and the entangled photons eventually allows us to prepare arbitrary pure polarization states. Moreover, owing to momentum conservation, entanglement between the momentum degrees of freedom can also be achieved using apertures to select well-defined momentum modes[16,21]; and the whole range of momentum states can be generated with neutral filters, phase plates and beam splitters that combine different momentum modes[17].

With these two degrees of freedom, the entire system of two photons has polarization states spanned by $|H\rangle_i$, $|V\rangle_i$ ($i = 1, 2$ labels the two photons) and momentum states spanned by $|a\rangle_i$ and $|b\rangle_i$. Upon identification of the momentum state $|a\rangle_i$ as the equivalent of the polarization state $|H\rangle_i$ and analogously for $|b\rangle_i$ and $|V\rangle_i$, one can prepare two copies of an arbitrary input state $|\Psi\rangle$, one stored in each degree of freedom.

As outlined above, the concurrence of $|\Psi\rangle$ is determined by the probability of observing the first photon in the antisymmetric state

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle|b\rangle - |V\rangle|a\rangle) \qquad (2)$$

where we have dropped the index '1', as from now on all considerations will concern only the first photon. As count rates rather than probabilities are accessible in laboratory experiments, we also need to count events corresponding to the detection of the remaining Bell states:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|b\rangle + |V\rangle|a\rangle) \qquad (3a)$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle|a\rangle \pm |V\rangle|b\rangle) \qquad (3b)$$

The probability P_A, which determines concurrence, is then given by the count rate for the observation of $|\psi^-\rangle$ normalized by the sum of the count rates for all four Bell states.

The central building block for this Bell-state measurement in our specific experimental set-up was a polarization-sensitive Sagnac interferometer containing two cylindrical lenses, as depicted in Fig. 1. The interferometer is used to perform a polarization-dependent rotation of the momentum modes, which is equivalent to a controlled-not (CNOT) operation. Input photons are first incident on a polarizing beam splitter, which transmits H-polarized photons and reflects V-polarized photons, so that H- and V-polarized photons propagate in opposite directions within the interferometer, and leave through the same exit port. An optical lens system, consisting of two 150 mm focal length cylindrical lenses, was rotated by 45° in the transverse plane with respect to the propagation direction of the H-polarized photons. The individual lenses were aligned at +45° and −45°. As each photon suffered three mirror reflections while propagating between the lenses, it was necessary to invert the orientation of one of them. The lenses were placed in a confocal arrangement, so that each lens was a distance of 150 mm from the double square aperture, and also 150 mm from the central mirror of the interferometer, forming a confocal imaging system with a magnification factor of one. The image formed by a cylindrical lens is inverted with respect to one transverse direction only, so that a lens aligned at ±45° forms an image that is rotated ±90° with respect to the object. Because H- and V-polarized photons counter-propagate within the interferometer, they encounter the lens oriented at different angles (±45°), so that the resulting image corresponding to H-polarized photons is rotated by 90°, while the image corresponding to the V-polarized photons is rotated by −90°, resulting in a relative difference of 180°. In the image plane, the Sagnac interferometer with the nested cylindrical lenses performs the so-called CNOT operation, where the momentum state evolves conditioned on the polarization—if the photon is vertically polarized, $|a\rangle$ evolves to $|b\rangle$ and vice versa; otherwise the momentum states remain unchanged. In reality, both the momentum states encounter an additional rotation by 90°, which can easily be accounted for by simply rotating our coordinate system. The operation of a similar CNOT gate was characterized in ref. 22. The crucial benefit of the CNOT operation is that it transforms the Bell states such that the momentum and polarization states factorize:

$$\mathrm{CNOT}(|\psi^\pm\rangle) = 1/\sqrt{2}(|H\rangle \pm |V\rangle)|b\rangle = |\pm\rangle|b\rangle \qquad (4a)$$

$$\mathrm{CNOT}(|\phi^\pm\rangle) = 1/\sqrt{2}(|H\rangle \pm |V\rangle)|a\rangle = |\pm\rangle|a\rangle \qquad (4b)$$

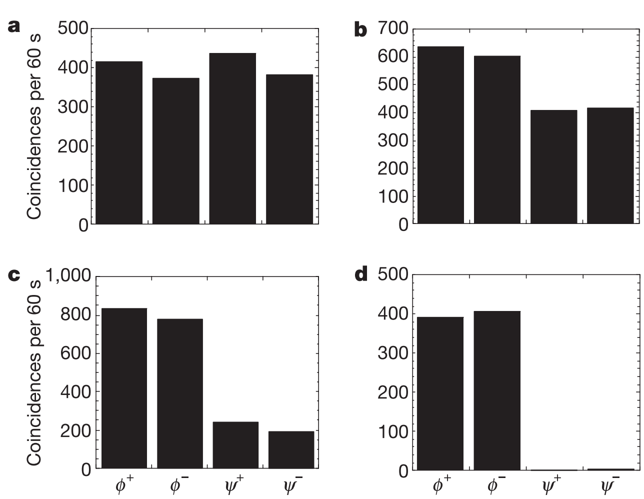

Figure 2 | Count rates. Experimentally obtained count rates of the Bell-state measurement (see equations (2) and (3)) on the twofold copy of input states $\alpha|01\rangle + \beta|10\rangle$ with **a**, $|\alpha| = 0.71 \pm 0.02$, **b**, $|\alpha| = 0.53 \pm 0.01$, **c**, $|\alpha| = 0.35 \pm 0.01$, and **d**, $|\alpha| = 0.99 \pm 0.03$.

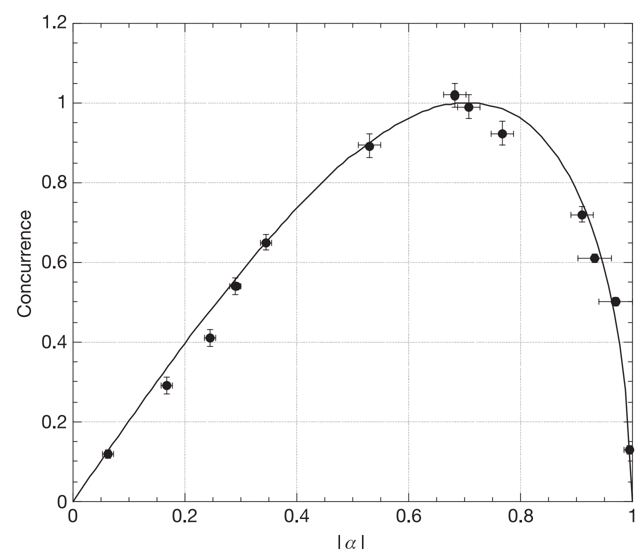

Figure 3 | Experimental values of concurrence. Data points, directly measured concurrence for states $\alpha|01\rangle + \beta|10\rangle$ as function of $|\alpha|$ with error bars due to poissonian statistics. The excellent agreement with the theoretical value $C = 2|\alpha|\sqrt{1-|\alpha|^2}$ (shown as the solid line) confirms the precision of the described measurement set-up.

Thus, observing a photon with $|-\rangle$ polarization and momentum $|b\rangle$ after the CNOT is equivalent to observing the state $|\psi^-\rangle$ before the CNOT operation, and analogously for the other Bell states. Thus, the final measurement simply consists of detecting $|\pm\rangle$ polarized photons in the modes a and b. This can easily be carried out with two detectors positioned in the paths of modes a and b, and additional half-wave plates and polarization analysers to discriminate the different polarizations. In particular, it should be emphasized that the four possible measurement results are the outcomes of a single measurement only.

In our experiment we measured the concurrence of states $\alpha|01\rangle + \beta|10\rangle$ with varying coefficients α and β. These states are particularly well suited, as the entire measurement protocol works perfectly, even for imperfect copies with different relative phases—which significantly relaxes the precison required during the preparation process. The coefficients α and β of the polarization and momentum degrees of freedom were varied by rotating the half-wave plate in the pump beam and shifting the apertures defining the momentum modes of photon 2, respectively. Figure 2 shows the experimental count rates for observations of the Bell states as defined in equations (2) and (3) for four different states with decreasing entanglement from Fig. 2a to Fig. 2d. Experimentally obtained concurrence C is depicted in Fig. 3 as a function of the varying coefficient α. The black dots show the experimentally obtained values, with error bars due to poissonian count statistics. The theoretical value of $C = 2|\alpha\beta| = 2|\alpha|\sqrt{1-|\alpha|^2}$ is plotted as a solid line and agrees virtually perfectly with the experimental observations. In particular, the maximum value $C = 1$ is obtained for $|\alpha| = 1/\sqrt{2}$, which provides additional experimental evidence for the purity of the input states.

Our work shows that it is possible to directly assess entanglement properties with few—in this case only one—local measurements. Whereas state reconstruction and subsequent mathematical determination of entanglement is a viable and successfully demonstrated option for systems with few constituents, more efficient approaches are required for large objects. Our present experiment gives a proof of principle that indeed it is possible to circumvent the highly inefficient state reconstruction, and reliably characterize the entanglement properties of an unknown quantum state.

Received 6 December 2005; accepted 27 January 2006.

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Acknowledgements We acknowledge support from the Brazilian agencies CNPq, CAPES, PRONEX, FUJB and FAPERJ. This work was performed as part of the Brazilian Millennium Institute for Quantum Information, and was supported by a fellowship within the postdoctoral programme of the German Academic Exchange Service (DAAD) as well as by a Feodor Lynen fellowship of the Alexander von Humboldt foundation. We are indebted to C. H. Monken for discussions.

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