Decoherence as phase diffusion

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We show that the decoherence of a superposition of two coherent states of the electromagnetic field in a cavity may be interpreted as a phase-diffusion process.

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I. INTRODUCTION

Since the beginnings of quantum mechanics, the classical limit of this theory has been the source of intense debate. Einstein considered a fundamental problem the ''inexistence at the classical level of the majority of states allowed by quantum mechanics,'' namely those involving the coherent superposition of two or more macroscopically separated localized states [1]. Schrödinger emphasized a similar point in his famous ''cat paradox'' [2]. The lack of nonlocal observables with matrix elements between those two localized states has been proposed as the reason for not observing interference effects stemming from the coherent superposition [3]. More recently, the role of decoherence in the quantum-classical transition has been emphasized [4]. The unavoidable interaction of the system under consideration (''small system'') with a reservoir produces, in a very short time (decoherence time), an entanglement between each of the distinct classical states of the small system and orthogonal states of the reservoir. This entanglement eliminates the interference between those classical states, for any measurement involving only observables of the small system (even if these observables are nonlocal). In this process, and as far as only the ''small system'' is concerned, the quantum superposition is turned into a statistical mixture, for which all the information on the system can be described in classical terms, so our usual perception of the world is recovered. Grasping this process is important not only for understanding the quantum-classical transition, but may eventually be useful for applications that require keeping coherence in mesoscopic or macroscopic systems, such as quantum computation [5].

Recent experiments with trapped ions [6] and cavity QED [7] have demonstrated the possibility of measuring the coherence between distinguishable localized states of a system, and, furthermore, have allowed the monitoring of the decoherence process in real time [7], thus allowing the testing of decoherence theories [4,8]. In these experiments, quantum superpositions of coherent states (associated either with the center-of-mass motion in the ion trap experiment or with the electromagnetic field in the cavity QED experiment) are generated. They can be represented in the following form:

$$\frac{1}{N_0}(|\alpha_0\rangle + e^{i\psi_0}|\alpha_0 e^{i\theta}\rangle), \tag{1}$$

where ψ_0 and θ are arbitrary constant phases and N_0 is the normalization factor. The quantity $d=2|\alpha_0\sin(\theta/2)|$ determines the distance between the two coherent states in phase space and is a measure of the macroscopicity of the system. The density matrix associated with Eq. (1) is

$$\hat{\rho}=\frac{1}{N_0^2}(|\alpha_0\rangle\langle\alpha_0|+|\alpha_0 e^{i\theta}\rangle\langle\alpha_0 e^{i\theta}|+e^{i\psi_0}|\alpha_0 e^{i\theta}\rangle\langle\alpha_0| + e^{-i\psi_0}|\alpha_0\rangle\langle\alpha_0 e^{i\theta}|). \tag{2}$$

The time-dependent behavior of this density matrix may be obtained by solving the corresponding master equation, after assuming a specific form for the coupling with the reservoir [4,8]. Quite generally, one shows in this way that the nondiagonal terms become negligibly small after a time of the order of $t_{\rm cav}/d^2$, where $t_{\rm cav}$ is the energy damping time of the system. When $d\gg 1$, and for times t such that $t_{\rm cav}/d^2 \ll t \ll t_{\rm cav}$ (the last restriction coming from the requirement that the two coherent states are still approximately orthogonal to each other), the density matrix of the system describes a classical statistical mixture:

$$\hat{\rho}\rightarrow\frac{1}{2}(|\alpha_0 e^{-\gamma t/2}\rangle\langle\alpha_0 e^{-\gamma t/2}|+|\alpha_0 e^{i\theta}e^{-\gamma t/2}\rangle\langle\alpha_0 e^{i\theta}e^{-\gamma t/2}|), \tag{3}$$

where γ is the energy damping rate.

A peculiar feature of the transition from Eqs. (2) to (3) is that it corresponds, when $|\alpha_0|\gg 1$ (so that $N_0^2=2$), to replacing α_0 by $\alpha_0\exp(-\gamma t/2)$ and randomizing the phase ψ_0 in Eq. (2). This heuristic procedure is sometimes given a physical interpretation, through the statement that the interaction of the system with the reservoir leads to the randomization of this phase. However, to the best of our knowledge, there has not been, up to now, an explicit derivation of this fact. Can decoherence be interpreted as the diffusion of a quantum phase? Would this interpretation be valid for all times, or only if the two coherent states are almost orthogonal? If this interpretation could be done, the state of the system would be represented at all times by the pure state

$$\frac{1}{N_t}(|\alpha\rangle + e^{i\psi}|\alpha e^{i\theta}\rangle), \tag{4}$$

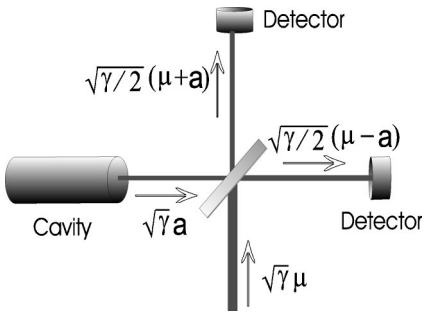

FIG. 1. Homodyne detection of the outgoing field.

where $\alpha=\alpha_0\exp(-\gamma t/2)$, and the phase ψ would be considered as a dynamical variable, evolving from its initial value through a diffusion equation. Here N_t is the normalization factor.

In this paper we show that it is indeed possible to derive a Fokker-Planck equation for the phase distribution, throughout the evolution of the state. This equation, which becomes a diffusion equation for $|\alpha|\gg 1$, corresponds to a particular realization of the system's evolution, under continuous monitoring of the reservoir. Since this monitoring represents a continuous retrieval of information on the interaction with the reservoir, the system remains in a pure state of the form (4), but the quantum phase undergoes a random walk process. This dynamical evolution is obtained by deriving a stochastic Schrödinger equation [9] corresponding to the assumed measurement scheme. The average over many realizations reproduces the results obtained from a master equation treatment.

It is not our purpose in this paper to propose a realistic experiment, but rather to show through a *gedanken* experiment that it is indeed possible to interpret the decoherence process as stemming from quantum phase diffusion, and, furthermore, that this interpretation must be associated with a specific realization of the process, corresponding to a continuous monitoring of the reservoir. In the next section we give a detailed description of the measurement scheme under consideration. In Sec. III we derive a Fokker-Planck equation for the quantum phase, and show that the results obtained from the master equation are retrieved. Our conclusions are summarized in Sec. IV.

II. A MODEL FOR THE CONTINUOUS MEASUREMENT OF THE RESERVOIR

We consider for definiteness that the state (1) corresponds to an electromagnetic field in a cavity, and take for simplicity the special case of Eq. (1), corresponding to $\theta=\pi$. We assume that the field losses are due solely to the transmission of the field outside the cavity. The transmission coefficient is $\sqrt{\gamma}$, $\gamma=1/t_{\rm cav}$ being the damping rate for the field intensity in the cavity. The field emerging from the cavity can be monitored by means of the balanced homodyne detection scheme shown in Fig. 1. The positive-frequency part of the outgoing field is associated with $\sqrt{\gamma}\hat{a}$, where $\hat{a}$ is the annihilation operator for one photon. This field is combined in a beam splitter with a classical field, the complex number $\sqrt{\gamma}\mu$ being associated with its positive-frequency part. We assume that this classical field is in phase quadrature with the coherent state $|\alpha_0\rangle$, and with the amplitude much larger than α_0, that is

$$\mu=ix\alpha_0, \quad x \text{ real}, x\gg 1. \tag{5}$$

At the beam splitter 50% of the field intensity is transmitted and 50% is reflected, so that the fields emerging from it to the detectors D_1 and D_2 are

$$\sqrt{\frac{\gamma}{2}}(\mu+\hat{a})\rightarrow D_1,$$

$$\sqrt{\frac{\gamma}{2}}(\mu-\hat{a})\rightarrow D_2. \tag{6}$$

In this way, the dissipation process is reduced to the detection of photons by any of the two detectors.

One should remark at this point that it is essential for our analysis to have the classical field in quadrature with the cavity field. Otherwise, a pure phase diffusion process would not be obtained, as the following discussion will show. The homodyne measurement allows a constant phase relation between the two fields, which would not be true for a heterodyne detection.

Before proceeding with the calculation of the evolution of the field state under this continuous monitoring, let us show that this process is consistent with a master equation of the Lindblad form [10], which may be written as

$$\dot{\hat{\rho}}=\frac{i}{\hbar}[\hat{\rho},H_0]+\mathcal{L}[\hat{\rho}(t)], \tag{7}$$

where $\hat{\rho}$ is the reduced density operator for the field inside the cavity, H_0 is the Hamiltonian governing the free evolution of the field mode,

$$\mathcal{L}[\hat{\rho}(t)]=\sum_m \left(\hat{C}_m\hat{\rho}\hat{C}_m^\dagger-\tfrac{1}{2}\{\hat{C}_m^\dagger\hat{C}_m,\hat{\rho}\}\right) \tag{8}$$

is the Lindblad operator, associated with the losses, and $\{.,.\}$ denotes an anticommutator. For a mode of the field interacting with a zero-temperature reservoir, one has only one operator $\hat{C}_1=\sqrt{\gamma}a$. Nevertheless, another operator may be introduced without changing the master equation: if μ is a complex number, we insert $\hat{C}_2=\sqrt{\gamma}\mu$ into Eq. (8). Clearly, the contribution from this c number vanishes identically, so the master equation remains invariant. We now replace the jump operators $\hat{C}_1$ and $\hat{C}_2$ by the two operators $\hat{C}_1'$ and $\hat{C}_2'$ defined by

$$\hat{C}_1'=\frac{\hat{C}_2+\hat{C}_1}{\sqrt{2}}=\sqrt{\frac{\gamma}{2}}(\mu+\hat{a}),$$

$$\hat{C}_2'=\frac{\hat{C}_2-\hat{C}_1}{\sqrt{2}}=\sqrt{\frac{\gamma}{2}}(\mu-\hat{a}). \tag{9}$$

This transformation, which also leaves the master equation invariant, allows one to associate a Lindblad operator with each detector in the homodyning scheme displayed in Fig. 1. In other words, the dissipation process can be described equivalently either through the direct absorption of photons from the cavity mode (operator $\hat{C}_1$) or through the detection of photons by the detectors D_1 and D_2 (operators $\hat{C}_1'$ and $\hat{C}_2'$). This implies that the continuous monitoring of the detections in the two detectors depicted in Fig. 1 corresponds to a particular unfolding of the master equation (7). Therefore, the average over all the realizations (corresponding to all the possible sequences of detections by D_1 and D_2) will necessarily reproduce the results stemming from the master equation [9]. We show in the next section that this unfolding corresponds to describing the decoherence process in terms of a quantum phase random walk. We also show explicitly that the master equation result is recovered after averaging over many realizations.

III. QUANTUM PHASE RANDOM WALK

We go back now to the description of the continuous monitoring scheme. We assume that at some given instant of time t the state of the system is (the interaction picture is used throughout)

$$|\Phi(t)\rangle = \frac{1}{\mathcal{N}_t}(|\alpha\rangle + e^{i\psi}|-\alpha\rangle), \tag{10}$$

with $\mathcal{N}_t^2/2 = 1+\cos\psi\exp(-2|\alpha|^2)$, and where ψ and α are functions of t. Apart from a normalization constant, the action of each operator $\hat{C}_m'$ on the state (10) may be written as

$$\hat{C}_1'|\Phi\rangle \rightarrow |\alpha\rangle + \frac{\mu-\alpha}{\mu+\alpha}e^{i\psi_0}|-\alpha\rangle,$$

$$\hat{C}_2'|\Phi\rangle \rightarrow |\alpha\rangle + \frac{\mu+\alpha}{\mu-\alpha}e^{i\psi_0}|-\alpha\rangle. \tag{11}$$

These actions stand for the quantum jumps (''clicks'' of the detectors) associated with the detection process [9].

On the other hand, between two detections, the amplitude of the field evolves under the nonunitary operator [9]

$$\exp\left(-t\sum_m \hat{C}_m'^{\dagger}\hat{C}_m'/2\right) = \exp(-\gamma|\mu|^2 t/2)\exp(-\gamma t\hat{a}^{\dagger}\hat{a}/2), \tag{12}$$

which is equivalent to $\exp(-\gamma t\hat{a}^{\dagger}\hat{a}/2)$ after renormalization. This amounts to setting

$$\alpha = \alpha_0\exp(-\gamma t/2). \tag{13}$$

The increase of the probability of having no photons in the cavity, resulting from this decay, is a consequence of the information gained from the fact that no detection is made [9].

The evolution of the field under continuous monitoring can thus be described as a smooth exponential decay of the coherent state amplitude interrupted by a succession of quantum jumps represented by the operators (9).

Because μ is in phase quadrature with α_0, and since $\alpha = \alpha_0\exp(-\gamma t/2)$, we may set

$$\frac{\mu-\alpha}{\mu+\alpha} = e^{i\epsilon}, \tag{14}$$

where ϵ is a real phase. Therefore, the action of the operators $\hat{C}_m'$ on the state (1) amounts to adding or subtracting a factor ϵ from the phase ψ_0. Since $|\mu| \gg |\alpha|$, it follows that $\epsilon \ll 1$. The phase ψ is no longer a constant, it is now a dynamical phase that evolves in time following a one-dimensional random walk with initial condition $\psi(0) = \psi_0$.

The photon detection rate for each of the two detectors is given by

$$dp_m/dt = \langle\Phi(t)|\hat{C}'^{\dagger}_m\hat{C}_m'|\Phi(t)\rangle \quad (m=1,2), \tag{15}$$

where $|\Phi(t)\rangle$ is given by Eq. (10).

From Eqs. (9), (10), and (15) it is easy to see that

$$dp_1/dt = \frac{\gamma}{2}|\mu|^2 + \frac{\gamma}{\mathcal{N}_t^2}[|\alpha|^2(1-\cos\psi e^{-2|\alpha|^2}) + \sin\psi e^{-2|\alpha|^2}(\mu\alpha^* - \mu^*\alpha)], \tag{16}$$

$$dp_2/dt = \frac{\gamma}{2}|\mu|^2 + \frac{\gamma}{\mathcal{N}_t^2}[|\alpha|^2(1-\cos\psi e^{-2|\alpha|^2}) - \sin\psi e^{-2|\alpha|^2}(\mu\alpha^* - \mu^*\alpha)]. \tag{17}$$

One should note that the detection rate in each detector is different and this difference is proportional to the sine of the phase difference between α and the classical field μ. If μ was in phase with the outgoing field α, the detection rate in both detectors would be the same.

In view of the results found in Eqs. (16) and (17), we may write the following equation for the probability $P(\psi,t)$ of having a certain value of the phase ψ at a given time $t+dt$:

$$\begin{aligned} P(\psi,t+dt) &= \langle\Phi(\psi-\epsilon,t)|\hat{C}_1'^{\dagger}\hat{C}_1'|\Phi(\psi-\epsilon,t)\rangle dt P(\psi-\epsilon,t) \\ &+ \langle\Phi(\psi+\epsilon,t)|\hat{C}_2'^{\dagger}\hat{C}_2'|\Phi(\psi+\epsilon,t)\rangle dt P(\psi+\epsilon,t) \\ &- \left[1 - \sum_m \langle\Phi(\psi,t)|\hat{C}_m'^{\dagger}\hat{C}_m'|\Phi(\psi,t)\rangle dt\right] P(\psi,t). \end{aligned} \tag{18}$$

Using Eqs. (15), (16), and (17) and $|\mu| \gg |\alpha_0|$, it is straightforward to obtain from Eq. (18) an equation for the time evolution of $P(\psi,t)$, after neglecting terms independent of x,

$$\frac{\partial P(\psi,t)}{\partial t}=\frac{\Gamma}{2}[P(\psi-\epsilon,t)+P(\psi+\epsilon,t)-2P(\epsilon,t)]+2\gamma x e^{-2|\alpha|^2}|\alpha_0|^2 e^{-\gamma t/2}\times\left[\frac{\sin(\psi-\epsilon)}{\mathcal{N}^2{}_{t,-\epsilon}}P(\psi-\epsilon,t)-\frac{\sin(\psi+\epsilon)}{\mathcal{N}^2{}_{t,+\epsilon}}P(\psi+\epsilon,t)\right], \tag{19}$$

where $\mathcal{N}_{t,\pm\epsilon}$ is the normalization factor of $|\Phi(\psi\pm\epsilon,t)\rangle$. In the equation above, we have introduced the variable Γ, given by

$$\Gamma=\gamma|\alpha_0|^2x^2, \tag{20}$$

which corresponds to the total jumping rate $dp_1/dt+dp_2/dt$ in leading order in x^2. On the right-hand side of Eq. (19) one may expand any function of $F(\psi\pm\epsilon)$ in the following way, since $\epsilon\ll 1$:

$$F(\psi\pm\epsilon,t)\simeq F(\psi,t)\pm\frac{\partial F(\psi,t)}{\partial\psi}\epsilon+\frac{1}{2}\frac{\partial^2F(\psi,t)}{\partial\psi^2}\epsilon^2+\mathcal{O}(\epsilon^3). \tag{21}$$

Replacing this expansion into Eq. (19) we find

$$\frac{\partial P(\psi,t)}{\partial t}=\frac{\epsilon^2\Gamma}{2}\frac{\partial^2P(\psi,t)}{\partial\psi^2}+4\epsilon\gamma x e^{-2|\alpha|^2}|\alpha_0|^2e^{-\gamma t/2}\frac{\partial}{\partial\psi}\left[\frac{\sin\psi}{\mathcal{N}_t^2}P(\psi,t)\right]. \tag{22}$$

Now, since $\epsilon\ll 1$,

$$e^{i\epsilon}\simeq 1+i\epsilon+\mathcal{O}(\epsilon^2)=\frac{ixe^{\gamma t/2}-1}{ixe^{\gamma t/2}+1}\simeq 1+\frac{2i}{xe^{\gamma t/2}}+\mathcal{O}[1/(x^2e^{\gamma t})], \tag{23}$$

and therefore $\epsilon\simeq 2e^{-\gamma t/2}/x$, that is, in this limit ϵ is twice the amplitude ratio between the decaying measured field and the classical field. This result is valid for all values of α_0 and for all times.

We have derived, therefore, a Fokker-Plank equation for the quantum-phase distribution function, with time-dependent diffusion and drift coefficients:

$$\frac{\partial P(\psi,t)}{\partial t}=2\gamma|\alpha_0|^2e^{-\gamma t}\frac{\partial^2P(\psi,t)}{\partial\psi^2}+8\gamma e^{-2|\alpha|^2}|\alpha_0|^2e^{-\gamma t}\frac{\partial}{\partial\psi}\left[\frac{\sin\psi}{\mathcal{N}_t^2}P(\psi,t)\right]. \tag{24}$$

In the limit of $|\alpha|\gg 1$, when the coherent states are approximately orthogonal to each other, the equation above describes a pure diffusion, as the drift term vanishes as $e^{-2|\alpha|^2}$. Equation (24) also shows that the diffusion coefficient becomes very small when $t\gg 1/\gamma$. In this limit the two coherent states strongly overlap, and the field in the cavity approaches the vacuum state. The process cannot be considered in this limit as purely diffusive.

In order to solve Eq. (24) and find a distribution for the probability of having a certain phase ψ in a given time we can do the following change of variables:

$$Q(\psi,t)=\frac{\mathcal{N}_0^2}{\mathcal{N}_t^2}P(\psi,t). \tag{25}$$

It is straightforward to see that, after this change of variable, we change Eq. (24) into a purely diffusion equation,

$$\frac{\partial Q(\psi,t)}{\partial t}=2\gamma|\alpha_0|^2e^{-\gamma t}\frac{\partial^2Q(\psi,t)}{\partial\psi^2}. \tag{26}$$

This equation can be solved by removing the time dependence of the diffusion coefficient through another change of variables. We introduce a new variable $T=f(t)$, such that $dT/dt=\exp(-\gamma t)$, which implies that $T=[1-\exp(-\gamma t)]/\gamma$ (choosing for simplicity the same time origin for T and t). The diffusion equation becomes then:

$$\frac{\partial Q(\psi,T)}{\partial T}=2\gamma|\alpha_0|^2\frac{\partial^2Q(\psi,T)}{\partial\psi^2}. \tag{27}$$

The well-known solution of this equation, subject to the initial condition $Q(\psi,0)=\delta(\psi-\psi_0)$, is given by

$$Q(\psi,t)=\frac{\exp[-(\psi-\psi_0)^2/8\gamma|\alpha_0|^2T]}{|\alpha_0|\sqrt{8\pi\gamma T}}, \tag{28}$$

which implies that we have, for the actual probability of finding a certain value of the phase ψ in a time t,

$$P(\psi,t)=\frac{\mathcal{N}_t^2}{\mathcal{N}_0^2}\frac{\exp\{-(\psi-\psi_0)^2/8|\alpha_0|^2[1-\exp(-\gamma t)]\}}{|\alpha_0|\sqrt{8\pi[1-\exp(-\gamma t)]}}. \tag{29}$$

We use now this result to calculate the density matrix corresponding to a statistical mixture of state (10) with distribution $P(\psi,t)$ for the phase ψ. Using Eq. (29) we get

$$\left\langle\frac{e^{i\psi}}{\mathcal{N}_t^2}\right\rangle=\int P(\psi)\frac{e^{i\psi}}{\mathcal{N}_t^2}d\psi=\frac{e^{-2|\alpha_0|^2(1-e^{-\gamma t})}e^{i\psi_0}}{\mathcal{N}_0^2} \tag{30}$$

and

$$\left\langle \frac{1}{\mathcal{N}_t^2} \right\rangle = \frac{1}{\mathcal{N}_0^2}, \tag{31}$$

so that we obtain for $\overline{|\Phi\rangle\langle\Phi|}$ the same expression which is found by solving explicitly the master equation for $\hat{\rho}$ [8],

$$\hat{\rho} = \frac{1}{\mathcal{N}_0^2}[|\alpha_0 e^{-\gamma t/2}\rangle\langle\alpha_0 e^{-\gamma t/2}| + |-\alpha_0 e^{-\gamma t/2}\rangle \times\langle -\alpha_0 e^{-\gamma t/2}| + e^{-2|\alpha_0|^2(1-e^{-\gamma t})}(e^{i\psi_0}|-\alpha_0 e^{-\gamma t/2}\rangle \times\langle\alpha_0 e^{-\gamma t/2}| + e^{-i\psi_0}|\alpha_0 e^{-\gamma t/2}\rangle\langle -\alpha_0 e^{-\gamma t/2}|)]. \tag{32}$$

IV. CONCLUSION

We have shown that for a quantum superposition of coherent states, it is indeed possible to interpret decoherence as the diffusion of the quantum phase between the two states, as long as the overlap of the two coherent states can be neglected.

Since such an interpretation amounts to considering the state as a pure state of the form (4), it is not surprising that it should correspond to a specific realization of the dissipation process, in which all the photons leaking out of the cavity are detected. We have shown indeed that a phase diffusion process occurs if there is a continuous monitoring of the leaking field via homodyne detection, with a local oscillator field which is in quadrature with the field to be measured.

Of course, this is by no means the only process leading to phase diffusion. Other reservoir models, in which the field leaking is replaced for instance by atomic reservoirs [11], could also lead to similar results. A simple example of an alternative procedure to the one described in this paper is obtained by replacing the homodyning field in Eq. (5) by another one which, while still in phase quadrature with the cavity field, is proportional to α instead of α_0. Of course all these different models for single realizations must lead to the same final expression (32). This multiplicity of equivalent realizations corresponds to the multiple possible ways of unfolding the master equation which describes the dissipation process. One should stress that by considering the evolution of the system under continuous measurement, one is able to represent the state of the system at each instant of time by a pure state, with a phase which undergoes a diffusion process described by a Fokker-Planck equation. The evolution of the system when no monitoring is made can also be described of course by a Fokker-Planck equation, obtained directly from the master equation (7), by introducing a phase-space representation of the density operator. In this case, however, the system cannot be represented by a pure state (cf. [12]).

One should note that the study of the evolution of a Schrödinger catlike state under a stochastic Schrödinger equation has also been undertaken by Garraway and Knight [13]. They have displayed a localization of the Schrödinger cat in one of the two coherent states, after some steps of the evolution, and also under homodyne detection of the outgoing field. The difference from our procedure is quite simple: those authors do not require the classical homodyning field to be in phase quadrature with the field in the cavity, as assumed in the present case. This fact prevents each ''click'' from being interpreted as a change in a quantum phase. On the other hand, their scheme leads to a change of the ''weight'' of each coherent state, which leads to the localization phenomenon in phase space. Our requirement of having the classical and the cavity fields in quadrature also implies that a heterodyne measurement is ruled out. This kind of measurement also leads to a quantum state diffusion equation [9,14], but it does not yield a pure phase diffusion like the present scheme.

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