Production and detection of highly squeezed states in cavity QED

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We propose simple experiments in cavity quantum electrodynamics leading to the generation and detection of highly squeezed states of the electromagnetic field, even in the presence of experimental constraints like dissipation, atomic velocity spread, and detection inefficiency. Our proposal gives an operational meaning to the rotational and translational widths of a squeezed state.

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I. INTRODUCTION

The ability to construct high-quality cavities in the microwave domain and to manipulate Rydberg atoms with huge dipole moments [1] has allowed the testing of fundamental aspects of quantum mechanics, such as different manifestations of the quantization of the photon number [2,3], decoherence [4,5], teleportation [6], quantum nondemolition measurements [7–10], quantum logic gates [10], and complementarity [11]. Quantum states of the electromagnetic field such as Schrödinger-cat-like states [4,5,9] and Fock states [12] have been actually built in recent experiments.

Several methods have been proposed to engineer quantum states of the electromagnetic field in a cavity, using a sequence of two-level atoms that are sent through a high-Q superconducting cavity and interact with its field, being detected afterwards by level-selective ionizing counters [13]. Before and after the cavity, the atoms may also interact with electromagnetic fields in low-Q cavities, used to manipulate its internal states. The interaction between the atoms and the field in the superconducting cavity may be resonant or dispersive. In the resonant case, the atom may exchange photons with the cavity, while in the dispersive regime, the atom plays the role of a refraction index in the cavity, leading to phase shifts in the field or tuning the cavity into resonance with an external source [14]. In both cases, quantum correlations between the field and the atomic internal state are created. It is possible to tailor the entanglement between the atom and the field by proper choices of the interaction parameters, such as the interaction time, the field strength, the detuning, etc. Detection of the atom in one of the two states then projects the field onto the desired state. For most of the procedures proposed so far, the probabilistic nature of quantum mechanics implies that the atom may be detected in an undesired state, leading to a state of the field different from the one that is sought. In some cases, it is possible to send another atom to correct the state of the field, depending on the result of the first measurement, thus implementing a feedback procedure [15]. Most often, one restarts the experiment, discarding the unsuccessful realization. In this case, we say that the experiment is based on the postselection of events [16]. This has two consequences. First, the probability of production of the desired state usually decreases as the number of atoms needed to engineer the field increases. Second, a high detection efficiency is required because, if it happens that an atom is lost, it may cause the field to become a statistical mixture of the desired state and undesired ones. Because of this and the fact that one wants the preparation time to be shorter than the dissipation time, a small number of required atoms is highly desired.

Recently a method for the construction of a generic superposition of coherent states of the electromagnetic field using dispersive interactions of atoms and photons in cavities was presented, and shown to lead to a good approximation for arbitrary states of the field [17]. It was shown that a superposition of even a small number of coherent states along a straight line or on a circle in phase space can approximate nonclassical states of the field with a high degree of accuracy. In that procedure, the number of coherent states in the superposition grows linearly with the number of detected atoms.

In this paper, we propose a method for the construction of a squeezed state [18] in a cavity, using as in Ref. [17] a superposition of coherent states. We show, however, that, within the realm of present-day cavity QED techniques, it is possible to devise a procedure by which the number of coherent states in the superposition grows *exponentially* with the number of detected atoms. Therefore, we are able to achieve high values of squeezing after a few atoms. We show that the probability of getting the desired state is still reasonably high. Furthermore, we take into account the effect of nonunity detection efficiency and the role of the dissipation, as well as the spread of the atomic velocity. By using realistic parameters, we show that this procedure is experimentally viable, and that it is rather insensitive to the detection efficiency (of the order of 50% in recent experiments [5]).

In the next section, we describe our method in detail, taking into account the possible experimental constraints mentioned before. In Sec. III, we explain how the squeezing properties of the field inside the cavity may be measured and show how this measure is related to phase-space representations of the corresponding density operator. Our proposal gives an operational meaning to two possible definitions of the width of a squeezed state: the rotational and the translational widths (cf. [19]). Our conclusions are summarized in Sec. IV.

II. GENERATION AND DETECTION OF SQUEEZED STATES

A. Method for squeezed-state generation

While several of the methods proposed in the literature could be used in principle to generate squeezed states of the

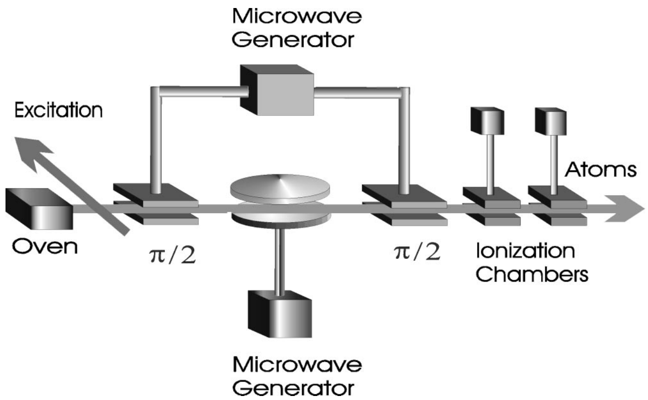

FIG. 1. Experimental scheme.

field, the procedure presented here has the advantage of simplicity, and of close connection with recently held experiments [2,5,10]. The basic experimental scheme is illustrated in Fig. 1. A high-Q superconducting cavity C is placed between two low-Q cavities (R_1 and R_2 in Fig. 1). The cavities R_1 and R_2 are connected to the same microwave generator. Another microwave source is connected to cavity C, allowing the injection of a coherent state in this cavity. This system is crossed by a velocity-selected atomic beam, such that an atomic transition $e \leftrightarrow g$ is resonant with the fields in R_1 and R_2, while another transition $e \leftrightarrow i$ is quasiresonant (detuning δ) with the field in cavity C. The relevant level scheme is shown in Fig. 2. Just before R_1, the atoms are promoted to the highly excited circular Rydberg state $|e\rangle$ (typical principal quantum numbers of the order of 50, corresponding to lifetimes of the order of some milliseconds). As each atom crosses the low-Q cavities, it ''sees'' a $\pi/2$ pulse, so that $|e\rangle \rightarrow [|e\rangle + |g\rangle]/\sqrt{2}$, and $|g\rangle \rightarrow [-|e\rangle + |g\rangle]/\sqrt{2}$. The atom interacts dispersively with the field in cavity C, so that transitions from levels e and g can be neglected, but there is a state-dependent energy shift of the atom-field system (Stark shift), which dephases the field, after an interaction time t_{int} between the atom and the cavity mode (the quantity t_{int} is actually an effective interaction time, which takes into account the Gaussian profile of the cavity mode). We assume that there is a dephasing of ϕ per

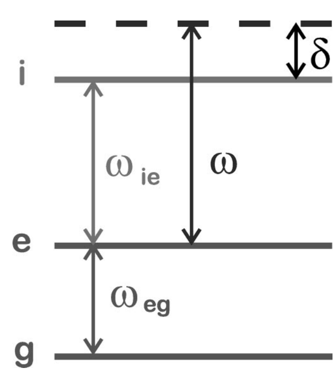

FIG. 2. Atomic level scheme. The transition $i \leftrightarrow e$ is detuned by δ from the frequency ω of a mode of cavity C, while the transition $e \leftrightarrow g$ is resonant with the fields in R_1 and R_2. State $|g\rangle$ is not affected by the field in C.

photon if the atom is in the state e, while there is no dephasing at all if the atom is in state g. This may be implemented through the level scheme displayed in Fig. 2: the cavity mode is close to resonance with a transition $i \leftrightarrow e$, but transitions from level g are sufficiently off resonant so that the corresponding energy shift is negligible. The one-photon phase shift is given by $\phi = (\Omega^2/\delta)t_{\mathrm{int}}$, where the Rabi frequency Ω measures the coupling between the atom and the cavity mode. Our scheme can also be easily applied to the situation in which both levels e and g suffer energy shifts.

The generation of a squeezed state of the field in C involves the following steps. One turns on the microwave source connected to cavity C for some time Δt, so that a coherent state $|\alpha\rangle$ is injected into the cavity. For definiteness, we assume that α is real. An atom in state $|e\rangle$ is then sent through the system. The velocity of the atom is chosen so that $\phi = \pi$. After the atom crosses R_1, C, and R_2, the entangled atom-field state $|\psi_{\mathrm{atom+field}}\rangle$ becomes

$$|\psi_{\mathrm{atom+field}}\rangle = \tfrac{1}{2}[|e\rangle(|-\alpha\rangle - |\alpha\rangle) + |g\rangle(|\alpha\rangle + |-\alpha\rangle)]. \tag{1}$$

Finally the internal state of the atom is detected by two field-ionization detectors (see Fig. 1). Upon detection of the atomic state, the state of the field is projected onto either a sum or a difference of two coherent states:

$$|\pm\rangle = \frac{1}{N_\pm}[\alpha\rangle \pm |-\alpha\rangle], \tag{2}$$

where $N_\pm^2 = 2 \pm 2\exp(-2\alpha^2)$. One should note that this first step is similar to the one realized by Brune *et al.* [5], where a superposition of two coherent states was also obtained (albeit not with the phase difference between the two coherent states equal to π).

Let us consider now the quadratures

$$\hat{X} = \frac{\hat{a} + \hat{a}^\dagger}{\sqrt{2}}, \tag{3}$$

$$\hat{Y} = \frac{\hat{a} - \hat{a}^\dagger}{i\sqrt{2}}, \tag{4}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators corresponding to the field mode in the cavity. The variance of the $\hat{Y}$ quadrature may be written as

$$(\Delta Y)^2 = \frac{1-\chi}{2},$$

where the squeezing parameter is defined by

$$\chi = \langle \hat{a}^2 + \hat{a}^{\dagger 2} - 2\hat{a}^\dagger \hat{a}\rangle - 2\langle \hat{Y}\rangle^2. \tag{5}$$

One should note that $\chi = 0$ for a coherent state and $\chi = 1$ for a Y-quadrature eigenstate. It is easy to show that the squeezing parameters corresponding to the states $|\pm\rangle$ are given by

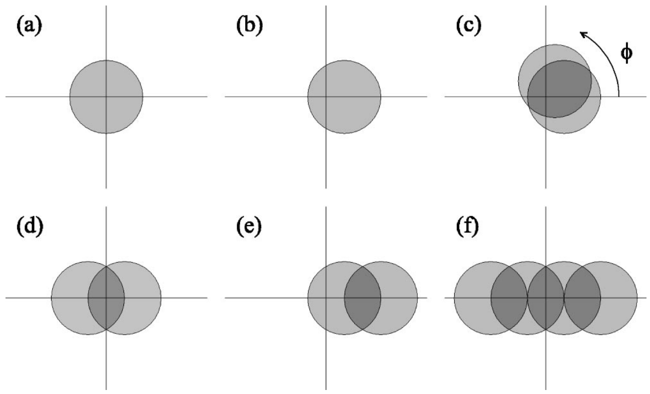

FIG. 3. Field evolution in cavity C during the construction of the squeezed state. (a) The field in the cavity starts in the vacuum state. (b) A coherent state $|\alpha\rangle$ is injected into the cavity. (c) An atom, prepared in a superposition of the states $|e\rangle$ and $|g\rangle$ in the first Ramsey zone, crosses the cavity; if the atom crosses cavity C in $|e\rangle$, the field is dephased by π. (d) If the atom is detected in $|g\rangle$ after the second Ramsey zone, the field in the cavity is collapsed onto a superposition of two coherent states, which exhibits squeezing. (e) The field is then displaced by 2α. (f) A second atom goes through the same system; if the atom is detected again in state $|g\rangle$, a higher-squeezed state is obtained.

$$\chi_{\pm}=\pm\frac{4\alpha^2 e^{-2\alpha^2}}{1\pm e^{-2\alpha^2}}, \tag{6}$$

so that the states $|-\rangle$ do not exhibit squeezing, while the states $|+\rangle$ exhibit a squeezing that is maximum for $\alpha\approx0.8$ (one attains then a squeezing of $\approx55.7\%$). The probability of getting the $|+\rangle$ state is equal to the probability of detection of the atom in the $|g\rangle$ state:

$$P_{g,1}=|\langle g|\psi_{\text{atom+field}}\rangle|^2=\frac{1}{4}N_+^2 .$$

The process of getting this squeezed state, starting from the vacuum state in the cavity, is illustrated in Figs. 3 (a)–3(d), where each coherent state in the superposition is represented by a circle. We see therefore that one may get a squeezed field in a cavity so long as one selects events for which the atom is detected in state $|g\rangle$. This occurs, for the value of α that maximizes the squeezing, with a probability of 63.9%. One should note that this class of states was studied by Schleich *et al.* [20].

In order to get higher amounts of squeezing, one proceeds in the following way. After measuring the atom in state $|g\rangle$, one turns on the microwave source S, so as to inject in the cavity a coherent field with amplitude 2α. The effect of the microwave source may be represented by the displacement operator $\hat{D}(z,z^*)=\exp(z^*\hat{a}-z\hat{a}^\dagger)$, where z is the complex amplitude of the injected field. This operator is, up to a phase, the evolution operator (in the interaction picture) corresponding to the interaction between the field and a classical current oscillating with the field frequency. Its action on a coherent state is given by

$$\hat{D}(z,z^*)|\alpha\rangle=\exp(z\alpha^*-z^*\alpha)|\alpha+z\rangle.$$

Note that the phase in the above equation is zero if z and α are both real or imaginary.

In this way, the field in the cavity gets displaced, leading to the configuration shown in Fig. 3 (e). One then sends a second atom through the cavity, prepared in the same way as the first one. If the second atom is again detected in the state $|g\rangle$, one generates the state of the field:

$$|\Phi^2\rangle=\frac{1}{\mathcal{N}_2^2}[|-3\alpha\rangle+|-\alpha\rangle+|\alpha\rangle+|3\alpha\rangle].$$

This state is represented in Fig. 3(f). If one wants to stop at this step, one must look for the value of α that maximizes squeezing. Otherwise, if this procedure is continued, one gets after detecting the Nth atom,

$$|\Phi^N\rangle=\frac{1}{\mathcal{N}_N}\sum_{n=1-2^{N-1}}^{2^{N-1}}|(2n-1)\alpha\rangle, \tag{7}$$

where

$$\mathcal{N}_N^2=2^N+\sum_{k=1}^{2^N-1}2(2^N-k)e^{-2\alpha^2k^2}. \tag{8}$$

The variation of the squeezing as a function of the amplitude α is displayed in Fig. 4, when the number of detected atoms is $N=1$, 3, 5, and 10. This figure clearly shows that the squeezing becomes quite insensitive to the choice of α as the number of detected atoms increases.

The probability of detection of N atoms always in the same state g is given by

$$P_N=\prod_{n=1}^{N}P_{g,n}=\left(\frac{1}{4}\right)^N\mathcal{N}_N^2 .$$

A better way to visualize the state is through the corresponding Wigner distribution [21], which can be written in the following way [22]:

$$W(z,z^*)=2\,\mathrm{Tr}[\hat{\rho}\hat{D}(z,z^*)e^{i\pi\hat{a}^\dagger\hat{a}}\hat{D}^{-1}(z,z^*)], \tag{9}$$

where the density operator $\hat{\rho}$ describes the state of the electromagnetic field in the cavity, and z is a complex amplitude in phase space.

The Wigner representation for the state with $N=3$ and $\alpha\approx0.7$ is plotted in Fig. 5. As its shape suggests, this state is highly squeezed along the y axes (the squeezing parameter χ in this case is close to 0.9). It is also clear from that figure that the Wigner function exhibits in this case negative values. In fact, the state given by Eq. (7) does not belong to the class of minimum-uncertainty states, and cannot be obtained from a coherent state by a simple scale transformation. For a general value of N, we get from Eqs. (5) and (7):

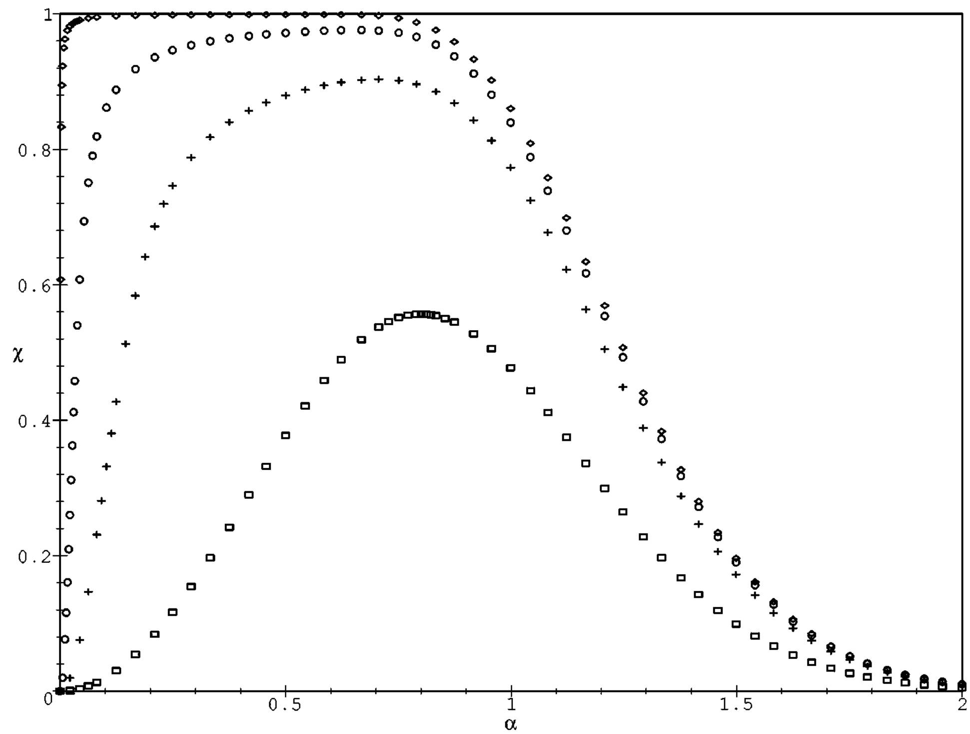

FIG. 4. Squeezing as a function of α when the number of detected atoms is $N=1$ (squares); $N=3$ (crosses); $N=5$ (circles), and $N=10$ (lozeanges). As the number of detected atoms increases, the squeezing becomes quite insensitive to the value of α.

$$\chi=\frac{8\alpha^2}{N_N^2}\sum_{k=1}^{2^N}(2^N-k)k^2e^{-2\alpha^2k^2}. \tag{10}$$

Given some N, one must choose the best value of α for getting maximum squeezing. Figure 6 displays the maximum squeezing, the best value for α^2, and the probability for constructing the state as a function of N. This result shows that squeezing reaches virtually 100% with a small number of detected atoms, while the state production probability is still large enough to make the experiment worthwhile. This can be explained by the fact that the number of coherent states composing these states is doubled by each detected atom. One should note that the limit of 100% squeezing corresponds to the construction of a quadrature eigenstate. We show in the Appendix that this limit is actually attained in the double limit $N\rightarrow\infty$, $\alpha\rightarrow 0$, with $2^N\alpha^2\gg 1$.

Our procedure leads thus to the construction of squeezed states of the electromagnetic field through the superposition of coherent states. An analogous decomposition has been considered before [23], in terms of a continuous integration. Our proposal leads instead to a discrete sum, and gives an operational meaning to such a decomposition.

All the calculations done above were made for an ideal experiment. A realistic analysis must contemplate the actual experimental conditions, and take into account the roles of dissipation, velocity spread, and detection efficiency.

B. Role of dissipation

We consider first the effect of dissipation. We start from the master equation for a field in contact with a zero-temperature reservoir:

$$\dot{\rho}=-\frac{\gamma}{2}(\rho a^\dagger a+a^\dagger a\rho-2a^\dagger\rho a). \tag{11}$$

One should note that for typical experimental setups [10], the average number of thermal photons is of the order of 0.1 or less, and therefore can be safely neglected in the present analysis.

From this equation, it follows that the evolution of the normal-order characteristic function [22],

$$C_N(\lambda,\lambda^*,t)=\mathrm{Tr}(\rho e^{\lambda a^\dagger}e^{-\lambda^* a}),$$

is given by [24]

$$C_N(\lambda,\lambda^*,t)=C_N(\lambda e^{-\gamma t/2},\lambda^* e^{-\gamma t/2},0). \tag{12}$$

From the definition of this characteristic function, it follows that

$$\langle (a^\dagger)^p a^q\rangle(t)=(-1)^q\frac{\partial^{p+q}}{\partial\lambda^p\partial\lambda^{*q}}C_N(\lambda=0,\lambda^*=0,t).$$

Thus, we have

$$\langle (a^\dagger)^p a^q\rangle(t)=e^{-(p+q)\gamma t/2}\langle (a^\dagger)^p a^q\rangle(0). \tag{13}$$

This relation, which is valid for any state of the electromagnetic field, implies that both the squeezing and the intensity decay at the same rate. Therefore, dissipation plays a much milder role here than in decoherence experiments [5].

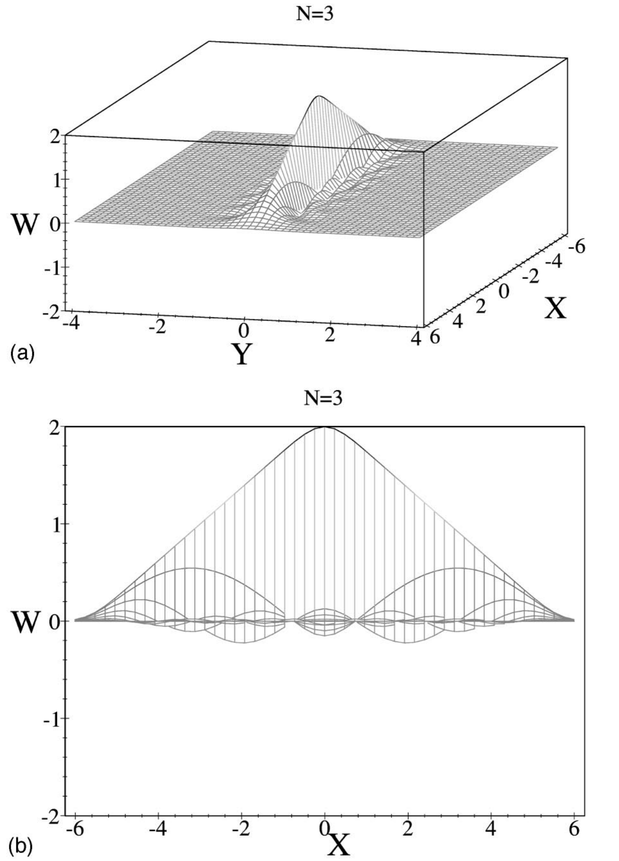

FIG. 5. (a) Wigner function for the state obtained after the detection of the third atom, with $\alpha \approx 0.3$. The high degree of squeezing is evident in the picture. (b) Projection of the same Wigner function on the x axis; the negative values signal the fact that this state is not obtained from a coherent state via a scale transformation.

C. Role of the atomic velocity spread

We now analyze the effect of the spread in the velocity of the atoms. The θ pulse at the cavities R_1 and R_2 and the phase shift ϕ at cavity C depend on the velocity of the atoms, since they are proportional to the time the atoms spend in each cavity. The state of the atom-field system right before atomic detection, and for arbitrary values of θ and ϕ, is given by

$$\left(\cos^2\frac{\theta}{2}|e^{i\phi}\alpha\rangle - \sin^2\frac{\theta}{2}|\alpha\rangle\right)\otimes|e\rangle + \cos\frac{\theta}{2}\sin\frac{\theta}{2}(|e^{i\phi}\alpha\rangle + |\alpha\rangle)\otimes|g\rangle. \tag{14}$$

It is clear that when the values of θ and ϕ are different from the prescribed $\theta = \pi/2$ and $\phi = \pi$, the state of the field is no longer that given by Eq. (7), and the squeezing may be spoiled.

The main effect is due to changes in the phase shift in cavity C. When $\phi \neq \pi$, the coherent states in the superposition are not aligned and therefore ΔY is increased. The fluctuations of θ act in second order and do not change the weights of the coherent states in the superposition, as long as the atom is detected in $|g\rangle$. Only the probability of production is affected (if the rotations in R_1 and R_2 are not equal, this is not true anymore, but the effect is still of second order). Figure 7 displays the squeezing achieved when the fluctuations of ϕ (or the velocity spread) are kept within 1%. Since the value actually obtained is different for each realization, the error bars represent the standard deviation of the squeezing after many attempts. They increase with N, while the mean value of the squeezing decreases compared to the ideal case. The behavior displayed in Fig. 7 implies that there is a compromise between velocity spread, reproducibility, and squeezing. It is clear from that figure that the best choice would be $N=3$ in this case, with $\approx 89.9 \pm 0.4\%$ of squeezing.

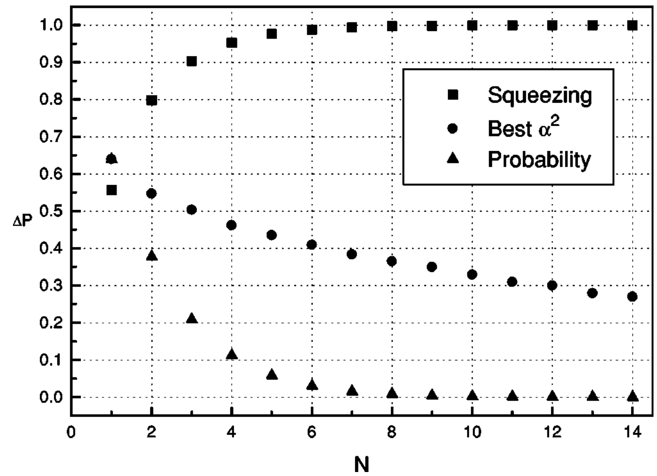

FIG. 6. Maximum squeezing, best choice for the square of the amplitude α of the initial coherent state, and the probability of formation of the squeezed states $|\Phi^N\rangle$ as a function of the number of atoms N.

D. Role of detection efficiency

The efficiency of the atomic counters available for this kind of experiment is at present around $40 \pm 15\%$ [5]. It

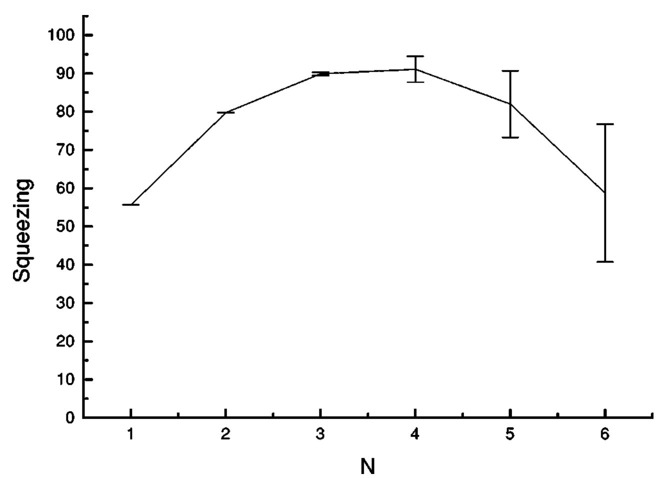

FIG. 7. Squeezing as a function of N, for a velocity dispersion of 1%. The error bars represent the standard deviation of the squeezing for several realizations.

would seem that a lost atom could change dramatically the state, since its construction is based on a postselection of the measured atomic states. Fortunately, this is not the case, as long as the field injection is conditioned to the detection of the atom. Let us consider the state just before the first atom is detected:

$$|\psi_1\rangle=\tfrac{1}{2}(|e_1\rangle|\Phi^-\rangle+|g_1\rangle|\Phi^+\rangle), \tag{15}$$

where $|\Phi^\pm\rangle=[|-\alpha\rangle\pm|\alpha\rangle])$. When the first atom is lost, if no field displacement takes place, the state after the second atom is

$$|\psi_2\rangle=\tfrac{1}{4}(-|e_2\rangle|e_1\rangle|\Phi^-\rangle+|g_2\rangle|g_1\rangle|\Phi^+\rangle). \tag{16}$$

There is therefore, as pointed out already in Ref. [9], a complete correlation between the state of the first atom and the subsequent ones (in the absence of dissipation). It is easy to see that the same property holds for any of the states $|\Phi^N\rangle$. Note also that the field will collapse in the same state as it would if the first atom had been detected. In the presence of dissipation, this remains true as long as the average time interval between detected atoms is much smaller than the field decoherence time.

III. MEASUREMENT OF SQUEEZING

We now address the problem of how to measure the squeezing of the field inside the cavity. We start by adapting to the level scheme under consideration a general procedure for measuring the Wigner function of the field [25], which is closely related to the one used above to generate the squeezed states. We show then that, even without making a full measurement of the Wigner function, it is possible to characterize the amount of squeezing of the state by means of simple measurements. Let $\hat{\rho}$ be the density operator of the field in the cavity, and suppose we turn on the microwave source connected to C, so as to inject a coherent field with complex amplitude z. As we have seen, this is equivalent to the action of the operator $\hat{D}(z,z^*)$ on the state of the field, that now becomes $\hat{\rho}'=\hat{D}(z,z^*)\hat{\rho}\hat{D}^{-1}(z,z^*)$.

We then send a probe atom through the same apparatus as before. We associate the dephasings suffered by the field in cavity C, and due to the dispersive atomic state-dependent interaction, to the unitary operator $\hat{\mathcal{T}}_e$ if the atom crosses cavity C in the state $|e\rangle$, and $\hat{\mathcal{T}}_g$, if it is in $|g\rangle$. We assume for the sake of generality that the field in R_2 is dephased by η from the field in R_1, so that in R_2 we have $|e\rangle\rightarrow[|e\rangle+\exp(i\eta)|g\rangle]/\sqrt{2}$ and $|g\rangle\rightarrow[-\exp(-i\eta)|e\rangle+|g\rangle]/\sqrt{2}$. It is then straightforward to show that the state of the atom+field system is given just before detection by

$$\begin{aligned}&\tfrac{1}{4}[|e\rangle\langle e|\otimes(\hat{\mathcal{T}}_e-e^{-i\eta}\hat{\mathcal{T}}_g)\hat{\rho}'(\hat{\mathcal{T}}_e^\dagger-e^{i\eta}\hat{\mathcal{T}}_g^\dagger)+|g\rangle\langle g|\\&\quad\otimes(\hat{\mathcal{T}}_g+e^{i\eta}\hat{\mathcal{T}}_e)\hat{\rho}'(\hat{\mathcal{T}}_g^\dagger+e^{-i\eta}\hat{\mathcal{T}}_e^\dagger)\\&\quad+(\text{terms nondiagonal in atomic space})].\end{aligned} \tag{17}$$

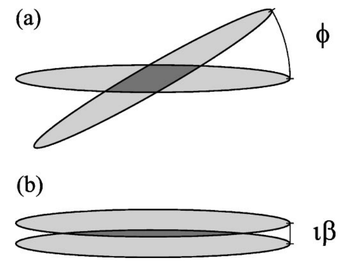

FIG. 8. The probability difference ΔP is sensitive to the overlap between the field distributions before and after interacting dispersively with a probe atom. (a) Rotational width can be measured by a rotation by ϕ. (b) Translational width is measured by a state-dependent displacement $i\beta$ of the field.

Note that the first step in the squeezed-state construction is recovered by setting $\hat{\rho}=|0\rangle\langle 0|$, $\eta=0$, $z=\alpha$, $\hat{\mathcal{T}}_g=\mathbf{1}$, and $\hat{\mathcal{T}}_e=e^{i\phi a^\dagger a}$, with $\phi=\pi$.

The atom is detected and the experiment is repeated many times, for each amplitude and phase of the injected field z, starting from the same initial state of the field $\hat{\rho}$. Finally, the probabilities P_e and P_g of detecting the probe atom in states e or g are determined. It is easy to show that

$$\begin{aligned}\Delta P&=P_g-P_e\\&=\mathrm{Re}\{e^{i\eta}\mathrm{Tr}[\hat{D}(z,z^*)\hat{\rho}\hat{D}^{-1}(z,z^*)\hat{\mathcal{T}}_g^\dagger\hat{\mathcal{T}}_e]\}.\end{aligned} \tag{18}$$

Expression (18) is very useful and leads to several interesting special cases. Choosing $\eta=0$, $\hat{\mathcal{T}}_g=\mathbf{1}$, $\hat{\mathcal{T}}_e=e^{i\pi a^\dagger a}$, and comparing the resulting expression with Eq. (9), we can see that

$$\Delta P=P_g-P_e=W(-z,-z^*)/2. \tag{19}$$

Therefore, the difference between the two probabilities yields a direct measurement of the Wigner function (one should note that, due to the fact that here $|g\rangle$ does not interact with the field in cavity C, this expression differs from the one given in Ref. [25]). Suppose now $\hat{\rho}=|\psi\rangle\langle\psi|$, $\eta=0$, and $z=0$ (no field is injected into the cavity); in this case, ΔP is the real part of the overlapping between the state $|\psi\rangle$ and the transformed state $\hat{\mathcal{T}}_g^\dagger\hat{\mathcal{T}}_e|\psi\rangle$. Let $\hat{\mathcal{T}}_g=\mathbf{1}$ and $\hat{\mathcal{T}}_e=e^{i\phi a^\dagger a}$: ΔP is now a measurement of the ''rotational width'' of the state $|\psi\rangle$, as shown in Fig. 8(a). The definition of the rotational width as a measure of squeezing was proposed in Ref. [20], and it is given an operational meaning here. Applying this method to the states $|\Phi^N\rangle$ given by Eq. (7), we can see that the larger N is, the faster ΔP will decrease as a function of ϕ, as shown in Fig. 9 (one may span several values of the dephasing ϕ by changing the atomic speed or the detuning δ). One should note that the measurement of the rotational width according to the above prescription amounts to the measurement of the values at the origin of phase space of a family of phase-space representations closely related to the Wigner function. As shown in Ref. [25], this family corresponds to an imaginary s parameter in the Cahill-Glauber characterization of phase-space representations [22].

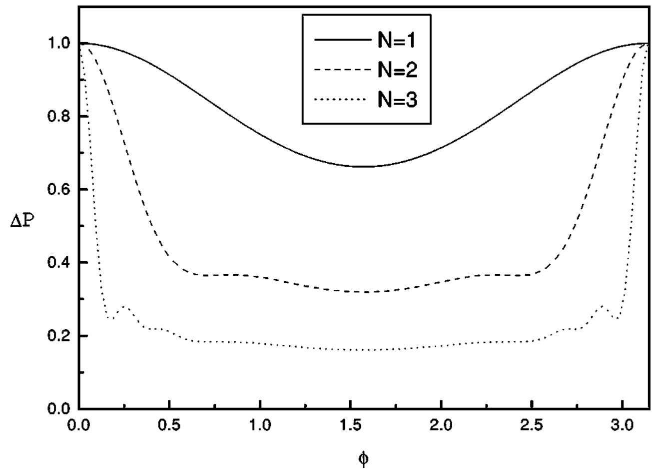

FIG. 9. ΔP as a function of the rotation angle ϕ for the rotational width measurement for the states $|\Phi^N\rangle$.

A different kind of measurement of squeezing can be obtained by setting $\hat{\mathcal{T}}_g = \mathbf{1}$ and $\hat{\mathcal{T}}_e = D(i\beta)$. One would then measure the intersection between the state and a translated version of it, yielding the ''translational width'' of the state. This may be achieved by using the quantum switch scheme proposed in Ref. [14]. In this case, the microwave source attached to cavity C is off resonant with respect to the cavity frequency, so that no field is injected into the cavity, when no atom is present. However, when an atom crosses the cavity in the state $|e\rangle$, the frequency of the cavity changes in such a way (due to the atomic refraction index) that it becomes resonant with the source field, which is then allowed into the cavity. On the other hand, when the atom crosses cavity C in the state $|g\rangle$, nothing happens, since as before this state does not interact with the cavity mode. Choosing β real would allow a measurement of the quadrature $\hat{Y}$ as depicted in Fig. 8(b). Applying this measurement to the states $|\Phi^N\rangle$, one verifies that ΔP decreases as a function of β faster for the states of larger N (higher squeezing)—see Fig. 10. Setting $\beta' = i\beta$, we have

$$\Delta P = -\mathrm{Re}\{e^{i\eta}\mathrm{Tr}[\hat{\rho}\hat{D}(\beta', \beta'^*)]\}, \tag{20}$$

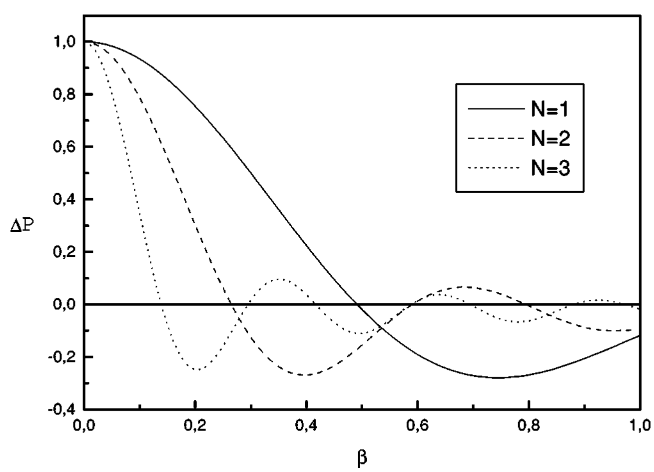

FIG. 10. ΔP as a function of the displacement β along the Y axis for the translational width measurement for the states $|\Phi^N\rangle$.

corresponding to the value of the symmetric-ordered characteristic function at the point β', which is given a physical interpretation here as a measure of the translational width of the state.

Before proceeding, let us consider the effect of the finite detection efficiency. If an atom is not detected after interacting with the cavity mode, the next atom will find a field described by the reduced density operator obtained from Eq. (17) by tracing out the atomic states: $\hat{\rho}'' = \frac{1}{2}(\hat{\mathcal{T}}_g\hat{\rho}'\hat{\mathcal{T}}_g^\dagger + \hat{\mathcal{T}}_e\hat{\rho}'\hat{\mathcal{T}}_e^\dagger)$. The value of ΔP for this second atom is then

$$\Delta P = -\tfrac{1}{2}\mathrm{Re}\{\mathrm{Tr}[(\hat{\mathcal{T}}_g\hat{\rho}'\hat{\mathcal{T}}_g^\dagger + \hat{\mathcal{T}}_e\hat{\rho}'\hat{\mathcal{T}}_e^\dagger)\hat{\mathcal{T}}_g^\dagger\hat{\mathcal{T}}_e]\}, \tag{21}$$

which reduces to Eq. (18), since $[\hat{\mathcal{T}}_g, \hat{\mathcal{T}}_e] = 0$. If the lost atom and the detected atom have different interaction times, due to velocity spread or field inhomogeneities, these operators may no longer commute. Such will be the case for the translation measurement, which will thus be spoiled. For rotations, however, the corresponding operators will still commute, and therefore the rotation measurements will be insensitive to these effects.

IV. CONCLUSIONS

The recent development of techniques for manipulating and measuring electromagnetic fields in high-Q superconducting cavities has led to fundamental tests of quantum mechanics and to the possibility of manufacturing and measuring nonclassical states of the electromagnetic field in cavities. We have shown in this paper that it is possible to realize experiments leading to the construction and detection of highly squeezed states of the electromagnetic field in a cavity. In particular, we have proposed a simple procedure to measure the rotational and translational width of these squeezed states, and have shown that the rotational width is closely related to the value of a generalized Wigner distribution at the origin of phase space, while the translational measurement is related to the corresponding characteristic function. In the method proposed here, a squeezing close to 100% is achieved with a small number of detected atoms, and with a state production probability still large enough to make the experiment worthwhile. Furthermore, our method is rather insensitive to the detection efficiency: the only requirement is that enough atoms are detected within the dissipation time.

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APPENDIX

We show in this Appendix that the squeezing parameter χ given by Eq. (10) goes to one in the double limit $N\to\infty$, $\alpha\to 0$, with $2^N\alpha^2 \gg 1$. From Eqs. (8) and (10), we have

$$\chi=\frac{8\alpha^2\sum_{k=1}^{2^N}(2^N-k)k^2e^{-2\alpha^2k^2}}{2^N+\sum_{k=1}^{2^N-1}2(2^N-k)e^{-2\alpha^2k^2}}. \tag{A1}$$

When $N\to\infty$ and $\alpha\to 0$, with $2^N\alpha^2\gg 1$, we have

$$\sum_{k=1}^{2^N}(2^N-k)k^2e^{-2\alpha^2k^2}\to 2^N\sum_{k=1}^{\infty}k^2e^{-2\alpha^2k^2} \tag{A2}$$

and

$$\sum_{k=1}^{2^N-1}(2^N-k)e^{-2\alpha^2k^2}\to 2^N\sum_{k=1}^{\infty}e^{-2\alpha^2k^2}. \tag{A3}$$

It follows from (A1), (A2), and (A3) that

$$\chi\to\lim_{\alpha\to 0}\frac{8\alpha^2\sum_1^{\infty}k^2e^{-2\alpha^2k^2}}{1+2\sum_1^{\infty}e^{-2\alpha^2k^2}} = \lim_{\alpha\to 0}-2\alpha^2\frac{\partial\ln\left(1+2\sum_{k=1}^{\infty}e^{-2\alpha^2k^2}\right)}{\partial(\alpha^2)}. \tag{A4}$$

The sum in the above expression can be calculated by using Poisson's formula,

$$\sum_{k=1}^{\infty}e^{-\alpha^2k^2}=\frac{1}{2}\left(\frac{\sqrt{\pi}}{\alpha}-1\right)+\frac{\sqrt{\pi}}{\alpha}\sum_{k=1}^{\infty}e^{-\pi^2k^2/\alpha^2}\to\frac{1}{2}\left(\frac{\sqrt{\pi}}{\alpha}-1\right), \tag{A5}$$

so that, finally,

$$\chi\to -2\alpha^2\frac{\partial\ln\left(\sqrt{\frac{\pi}{2\alpha^2}}\right)}{\partial(\alpha^2)}=1. \tag{A6}$$

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