Quantum Switches and Nonlocal Microwave Fields

L. Davidovich,* A. Maali, M. Brune, J. M. Raimond, and S. Haroche
Laboratoire de Spectroscopie Hertzienne de l'Ecole Normale Supérieure, Paris, France
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A scheme to realize an optical switch with quantum coherence between its "open" and "closed" states is presented. It involves a single atom in a superposition of circular Rydberg states crossing a high Q cavity. A combination of switches could be used to prepare a quantum superposition of coherent microwave field states located simultaneously in two cavities. Such nonclassical states and their decoherence due to cavity dissipation could be studied by performing atom correlation experiments.

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Logical circuits are based on the classical assumption that a gate must be either opened or closed. It is possible, however, to envision optical "quantum switches" (QS) which obey a different logic, being at the same time in a superposition of "open" and "closed" states. The fields controlled by these switches would be described as a superposition of quantum states with macroscopically distinguishable features. These states are often referred as "Schrödinger cats" in the literature [1,2]. In this Letter, we describe a simple QS consisting of a single atom prepared in a superposition of different energy states and sent across a high Q cavity coupled to a classical radiation source. This switch prepares a quantum superposition of the vacuum field with a classical coherent state. A combination of two QS can be used to build nonlocal field states occupying simultaneously two cavities. Nonlocal quantum states generally involve single particles located at two different points (two slits of a Young apparatus), or pairs of particles, as in the EPR experiment [3]. The sharing of a single photon between two separate cavities has also been discussed [4]. The field states considered here are quite different. They involve relatively large numbers of photons making up coherent fields described by classical parameters (amplitude and phase). Once prepared, these states could be analyzed by sending a second atom through the same QS combination and detecting, in the transition probabilities of this atom, interference effects sensitive to the field nonlocal coherence.

"Schrödinger cats" are very sensitive to dissipation and are expected to turn rapidly into mere statistical mixtures obeying classical logic [5,6]. Realizing QS devices is thus related to the observation of long lived quantum mechanical coherences between large physically separated subsystems. We discuss the orders of magnitude of realistic experiments with circular Rydberg atoms coupled to microwave superconducting resonators and conclude that QS's could be practically operated with fields containing tens of photons, over periods of time as long as 10 ms.

Let us discuss first a simple classical situation. Consider a high Q cavity (resonant frequency ω_c), coupled to a monochromatic source (frequency ω_s), with a detuning $\Delta=\omega_s-\omega_c$ much larger than the cavity bandwidth $\omega_c/Q=1/t_c$. The off-resonant source does not feed photons into the cavity. A nonabsorbing dielectric slab with a refractive index is now inserted into the cavity during a time t_i, momentarily tuning source and cavity into resonance. A classical coherent field is thus fed into the cavity with an amplitude proportional to t_i (if $t_i \ll t_c$). It subsists during a time of the order of t_c after the slab removal.

Similar experiments have been recently performed with the dielectric slab being replaced by a microscopic atomic sample [7]. The passage of the atom(s) through the cavity provided an index change large enough to tune the cavity into resonance with a source and to control the field flowing through the resonator. An atom has an interesting feature lacking in a classical slab: it is a quantum object which can be prepared in a superposition of different states. Since the atomic index depends upon the state, it is possible to realize a "medium" in a quantum superposition of states with different indices. We have already proposed to use this effect in order to split a field present in the cavity prior to the atom injection into two phase components, realizing a superposition of classical fields with different phases [6].

We consider now an initially empty cavity in which an atom is injected in a superposition of two states e and g, one of which, by its interaction with a third level, tunes the cavity into resonance with the source S [Fig. 1(a)]. After exiting the device, it is detected with a state sensitive counter (for example a set of two field ionization detectors ionizing in turn levels e and g). The state $|e\rangle$ is coupled to a third, more excited state $|i\rangle$ by a transition at frequency $\omega_0=\omega_c-\delta$ [Fig. 1(b)]. The coupling of the $e\rightarrow i$ transition to the cavity mode is characterized by the vacuum Rabi frequency Ω. We assume here that the coupling is adiabatically switched on (off) as the atom enters (exits) the cavity and remains constant in between. Taking into account a more general dependence complicates the algebra without changing the basic physics. The coupling is slightly nonresonant and an atom in level e inside the cavity pulls the mode frequency by Ω^2/δ (provided the field amplitude remains small enough; see more detailed discussion below). Adjusting the detunings so that $\Delta=\Omega^2/\delta$ ensures that an atom in level e tunes the cavity into resonance with the source. The atom, non-

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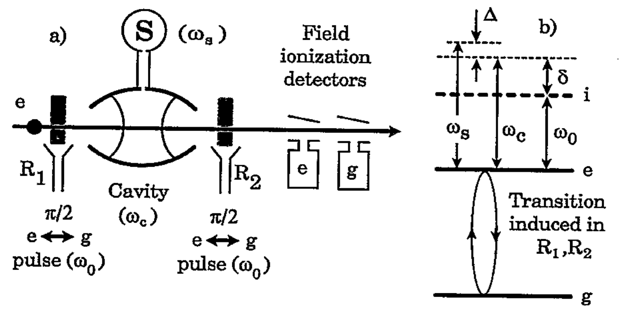

FIG. 1. (a) Sketch of a QS experiment. (b) Relevant atomic level scheme (detunings Δ and δ not to scale).

resonant with the field, then exits the cavity without undergoing any transition (adiabatic approximation). No frequency pulling occurs if the atom crosses the cavity in level g (ω_c very different from frequencies of all transitions originating from this level). The cavity-source detuning remains then Δ and the field cannot build up. Assume now that an atom is injected in the cavity in the $(|e\rangle+|g\rangle)/\sqrt{2}$ superposition. This is achieved by sending the atom (initially in $|e\rangle$) through a zone R_1 in front of the cavity where a classical microwave pulse performs a resonant $\pi/2$ pulse on the $e\rightarrow g$ transition. The system now ends up in the entangled state: $|\Psi_1\rangle=(|e;\alpha\rangle+|g;0\rangle)/\sqrt{2}$. The first and the second symbol in each ket refer to the state of the atom and field, respectively. 0 represents the vacuum and α the complex amplitude of the coherent field resonantly fed into the cavity during the time t_i the atom crosses it. The field is expressed in the cavity rotating frame and thus α does not depend upon time, as long as cavity relaxation is neglected. Detecting the atom at cavity exit would yield two possible outcomes, each leaving the field in one of two mutually exclusive states. A last important ingredient is required to turn this device into a QS. Before detection, the atom crosses a second zone R_2, identical to the first one, which again mixes $|e\rangle$ and $|g\rangle$. After R_2, the system has evolved into

$$|\Psi_2\rangle=\tfrac{1}{2}(|e;\alpha\rangle+|g;\alpha\rangle+|g;0\rangle-|e;0\rangle) \tag{1}$$

and a subsequent detection of the atom in $|e\rangle$ or $|g\rangle$ leaves now the field into one of the two coherent superpositions

$$|\Psi_f\rangle=\frac{1}{N_\pm}(|\alpha\rangle\pm|0\rangle) \tag{2}$$

with the sign + or − if the atom is detected in $|g\rangle$ or $|e\rangle$, respectively. $N_\pm=\{2[1\pm\exp(-|\alpha|^2/2]\}^{1/2}$ is a normalization factor reducing to $\sqrt{2}$ when $|\alpha|\gg1$. The final field state thus appears as a quantum superposition of cavity "filled" and "empty" states, clearly a "Schrödinger cat" situation.

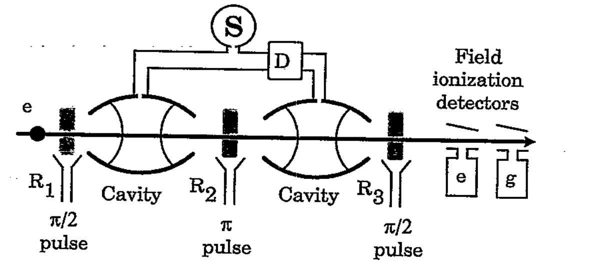

FIG. 2. Sketch of the double QS experiment preparing a nonlocal field belonging simultaneously to two cavities.

We discuss now how QS devices can be combined together to prepare nonlocal field states. Figure 2 shows a simple scheme involving two identical cavities coupled to the same source. An atom traverses successively both cavities and $\pi/2$ pulses are applied before and after the cavities (R_1 and R_3). A π pulse is applied in R_2 between the two cavities exchanging $|e\rangle$ and $|g\rangle$ and the QS "open" and "closed" states ($|e\rangle\rightarrow|g\rangle;|g\rangle\rightarrow-|e\rangle$). When the atom crosses the second cavity in level $|e\rangle$, the source injects the field $\alpha e^{-i\Delta t}$, $\Delta\times t$ being the phase shift between the cavity mode and the source during the atom time of flight t between the two resonators. This not essential phase can be compensated by inserting a dephaser D between S and the second cavity. We assume in the following that this compensation is performed.

A simple analysis shows that, after atomic detection, the field is given by

$$|\Psi_f\rangle=\frac{1}{N'_\pm}(|\alpha;0\rangle\pm|0;\alpha\rangle) \tag{3}$$

(+ and − signs when atom is found in e or g, respectively). The first and second symbol in each ket now refer to the state of the field in the first and second cavity, respectively. $N'_\pm$ is a normalization equal to $\{2[1\pm\exp(-|\alpha|^2)]\}^{1/2}$. Equation (3) describes a quantum coherence between two identical field states which differ by their location in separate cavities.

This coherence can be read out by sending a second atom through the apparatus after a delay T and measuring the probability of detecting it in $|e\rangle$ or $|g\rangle$ [8]. The second atom crosses the system under the same conditions as the first. This atom operates as a second QS, adding, when in its open state, the field $\alpha e^{-i\Delta T}$ into the system and dephasing an already present field by Δt_i. Each cavity ends up either empty or containing one of the three fields α, $\alpha e^{-i\Delta T}$, or $\alpha(e^{-i\Delta t_i}+e^{-i\Delta T})$. Before detection of the second atom, the state of the system has become

$$|\Psi_{\text{atom 2+field}}\rangle=\frac{1}{2N'_\pm}\{[|\alpha(e^{-i\Delta t_i}+e^{-i\Delta T});0\rangle+e^{i\varphi_1}|\alpha e^{-i\Delta T};\alpha\rangle](|g\rangle-|e\rangle)$$
$$-[|\alpha;\alpha e^{-i\Delta T}\rangle+e^{i\varphi_1}|0;\alpha(e^{-i\Delta t_i}+e^{-i\Delta T})\rangle](|g\rangle+|e\rangle)\}\,. \tag{4}$$

We have made the notation change $\pm 1 = \exp(i\varphi_1)$ with $\varphi_1 = 0$ or π. This phase is determined by the result of the first atom detection ($\varphi_1 = 0$ or π when it is found in e or g). The probabilities of counting the second atom in $|e\rangle$ or $|g\rangle$ are proportional to the square of the norm of the field states multiplying these kets in Eq. (4). Since these probabilities depend on φ_1, there is a correlation between the two atom detection outcomes. We are thus computing now the conditional probability $P_{a_1;a_2}(T)$ that an atom is found in a_2 a delay T after a first one has been detected in a_1 (a_1, a_2 stand for any combination of e and g). Using the known expression for the overlap of coherent states, we obtain

$$P_{a_1;a_2}(T) = \tfrac{1}{2} \pm \tfrac{1}{4}\cos(\varphi_1)\{\exp[-4|\alpha|^2\cos^2\Delta(T-t_i)/2] + \exp(-4|\alpha|^2\sin^2\Delta T/2)\} \tag{5}$$

(+ and − signs correspond to atoms detected in same or different states, respectively). In order to keep the result simple, we assume here $|\alpha| \gg 1$. A more general formula, valid for small fields α can be established easily.

Figure 3(a) represents (solid line) the probability $P_{e;e}(T) = P_{g,g}(T)$ as a function of the delay T between the two atoms (for a mean photon number $|\alpha|^2 = 10$ and $\Delta t_i = 7\pi/2$). $P_{e;e}(T)$ presents "peaks" equal to $\frac{3}{4}$ when $\Delta(T-t_i)/\pi$ is an odd integer or $\Delta T/\pi$ an even one, and decreases to a $\frac{1}{2}$ "background" for other T values. The width of all peaks is inversely proportional to $|\alpha|$. The probability of detecting the atoms in different levels is $P_{e;g}(T) = P_{g;e}(T) = 1 - P_{e;e}(T)$ [dashed line in Fig. 3(a)]. Experimentally, these curves could be obtained by sampling the results of a large number of double atom counts with a variable delay T, the first atom being always sent in an empty cavity system, obtained by waiting long enough for the field in both cavities to relax to the vacuum. Note that the first peaks are not observable, since T must be larger than the time of flight of the first atom through the apparatus.

The peaks in the $P_{a_1;a_2}(T)$ signal are related to a quantum interference, as revealed by a mere inspection of Eq. (5). If one could tune the phase φ_1 of the quantum coherence generated by the first atom, the amplitude of the peaks would be modulated and the corresponding probabilities would oscillate between $\frac{3}{4}$ and $\frac{1}{4}$. The probabilities we can actually measure correspond to two points in the "fringe signal" ($\varphi_1 = 0$ and π). If on the other hand the field were left by the first atom in a statistical mixture instead of a "cat" state, we could describe it by randomizing φ_1 in the above calculation, i.e., replacing $\cos(\varphi_1)$ by zero. The peaks would then disappear and the probability of detecting the second atom in either level would take the constant $\frac{1}{2}$ value, a result which can be interpreted by classical probability arguments.

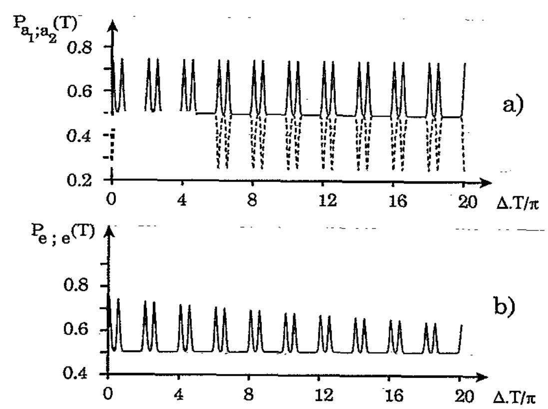

FIG. 3. (a) Probabilities $P_{e;e}(T)$ (solid line) and $P_{e;g}(T)$ (dashed line) to detect the first and the second atom in the same or in different states, respectively, as a function of the delay T between the two atoms ($|\alpha|^2 = 10$, $\Delta t_i = 7\pi/2$). Cavity relaxation neglected. (b) $P_{e;e}(T)$ with cavity relaxation included ($1/t_c = \Delta/1000$).

Quantum interferences are related to indistinguishable paths leading from an initial to a final state. Here, the path followed by the atoms could be traced by looking at the field left behind in the cavities. If these field states are distinguishable, i.e., orthogonal, the corresponding amplitudes do not interfere. This happens when $\cos^2[\Delta(T-t_i)/2]$ and $\sin^2[\Delta T/2]$ in Eq. (5) are different from zero, all field states in Eq. (4) being then different. When $\Delta(T-t_i) = (2k+1)\pi$, however, the first and fourth field states in Eq. (4) reduce to $|0;0\rangle$. As a result, two indistinguishable channels correspond to the same double atom count event and a quantum interference effect changes the probability from $\frac{1}{2}$ to $\frac{3}{4}$ or $\frac{1}{4}$. In the same way, the interference corresponding to the second and third field states in Eq. (4) account for the $\Delta T = 2k\pi$ peaks. Note that for $\Delta t_i = (2k'+1)\pi$, the two sets of peaks merge ($\Delta T = 2k\pi$) and the interference contrast becomes 100% (signal varying form $\frac{1}{2}$ to 1 or 0).

We have neglected so far field relaxation. In fact, the field energy is damped with the time constant t_c. The coherence between the two parts of the Schrödinger cat (in the one and two cavity cases) decays much faster [6]. Qualitatively, the quantum phase φ_1, which takes the value 0 or π when the first QS is operated, is scrambled by the dissipative coupling to the cavity walls and becomes a random quantity equally distributed between 0 and 2π after a delay of the order of $t_c/|\alpha|^2$. This has the effect of reducing the height of the interference peaks corresponding to increasing T values. The curve in Fig. 3(b) shows $P_{e;e}(T)$ when relaxation is included (details of the calculation will be presented elsewhere; Δ is chosen to be $1000\omega_c/Q$).

We conclude by discussing the feasibility of a practical experiment to demonstrate these nonlocal field effects. Three important conditions for operation of the QS are

$$t_i < t_c/|\alpha|^2, \quad \Omega|\alpha|/\delta \ll 1, \quad |\alpha|/(\Delta t_i) < 1\,. \tag{6}$$

The first one ensures that the quantum coherence does not appreciably relax during the opening time of the QS (and during the atom flight time across the apparatus, which is of the same order of magnitude). The second condition is required for the atomic index to be field-independent and to ensure the purely dispersive character of the atom-field interaction. The last condition (6) finally ensures that the "closed" state of the QS leaves the cavity essentially empty. The ratio of the amplitudes fed into the cavity when the QS is open and closed is indeed equal to Δt_i (fields proportional to t_i and $1/\Delta$, respectively). Equations (6) combined with the condition $\Delta = \Omega^2/\delta$ lead to two important limitations for α:

$$|\alpha|^2 \ll \Omega t_i, \quad |\alpha|^4 \ll \Omega t_c . \tag{7}$$

Equations (7) show that it is critical to achieve very large atom-field couplings Ω and to be able to make observations on very long time scales, with very weakly damped field and atomic systems. Circular Rydberg atoms [9] coupled to superconducting microwave cavities [6] are the most promising systems for these experiments. Couplings Ω of the order of 10^5 s^{-1} together with cavity damping times in the 10^{-1} s range make it possible to achieve $\Omega t_c = 10^4$ and to realize a QS controlling fields made of large absolute photon numbers. Circular states with principal quantum number $n \simeq 50$ and maximum angular momentum $l = n-1$ are required because they have long radiative lifetimes ($t_{\rm rad} = 3 10^{-2}$ s for $n=50$) and can survive in the e/g superposition during the time the atom interacts with the apparatus. Assuming that e, i, and g are circular levels with $n=50$, 51, and 49 respectively, the frequency $\omega_0/2\pi$ is 51.1 GHz (wavelength 6 mm) and the cavity tuned at $\omega_c \simeq \omega_0$ has a size of the order of 1 cm. An atom traveling at 10 m/s crosses the cavity in $t_i \simeq 10^{-3}$ s and reaches the state selective field ionization detector in a couple of milliseconds, a time much shorter than $t_{\rm rad}$. To summarize, conditions (6) and (7) are fulfilled for photon numbers up to $|\alpha|^2 \simeq 30$ with $1/t_c = 10$ s^{-1}, $7\pi/t_i = 2\times 10^4$ s^{-1}, $\Delta = 10^4$ s^{-1}, $\Omega = 10^5$ s^{-1}, and $\delta = 10^6$ s^{-1}. The curve in Fig. 3(b) corresponds to this choice of parameters.

Two experimental conditions must furthermore be met. First, the velocity of both atoms should be the same with an accuracy of $\simeq 1\%$, so that the two successive QS introduce the same amplitude field in the cavities, a condition required for the interference peaks to be observable. Laser cooling techniques can be used to slow down the atoms and to select their velocity with this precision, prior to their preparation in the circular state e. The atom counters must also have a high detection efficiency, since any "unread" atom will decrease the quantum coherence.

We have described a practical quantum switch able to control the flow of relatively large classical fields in one or two cavities, making it possible to realize nonlocal microwave field quantum states. The study of these states by double atom count measurements would permit the observation of quantum decoherence in a simple text book situation, and the monitoring "in real time" of the system evolution from quantum to classical behavior. Such an experiment could also be related to a crucial issue in measurement theory [10]. The field in the double cavity can indeed be considered as a "macroscopic pointer" whose position measures the internal microscopic state of the atom crossing the system. The decoherence of the "observable plus pointer" states in an essential stage of the measurement, which obviously occurs on the same time scale as the relaxation of the macroscopic pointer coherence studied in this proposed experiment.

Laboratoire de Spectroscopie Hertzienne de L'Ecole Normale Supérieure is associé au CNRS et à l'Université Pierre et Marie Curie.

*Permanent address: PUC University, Rio de Janeiro, Brazil.

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