Effect of atom pairs on the vacuum trapping state in micromasers: A Monte Carlo wave-function approach

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We calculate the lifetime of the vacuum trapping state in a micromaser, using both the Monte Carlo wave-function method and an analytic approximation, and show it to be a rapidly changing function of the average number of atoms in the cavity, when this number is much smaller than one. Our method leads to a realistic simulation of experiments, allowing for different field profiles, temperatures, and atomic velocity spreads. [S1050-2947(96)09509-1]

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Cooperative atomic effects play a minor role in usual lasers [1]. The dynamic and quantum behavior of these devices is well described by theories based on one-atom Hamiltonians, the cavity field being built by the incoherent sum of the individual atomic contributions [2]. Three notable exceptions should be mentioned, however. Under some conditions, it is possible to conceive a superradiant laser [3]. Also, cooperative effects seem to play an important role in a recent microlaser experiment [4], where laser oscillation has been demonstrated for a beam of ^{136}Ba atoms traversing a single-mode cavity, with less than one atom on the average inside the resonator. Indeed, one-atom theories predict a field intensity appreciably smaller than the observed one [4]. This may be accounted by the fact that the Poissonian distribution of the atoms in the beam leads to a probability of 26% for having two atoms inside the resonator, when the average number of atoms inside the cavity is equal to one. Finally, trapping states in micromasers [5,6] are highly sensitive to cooperative effects [7,8]. These states are predicted to occur in a high-Q cavity crossed by a monokinetic beam of excited two level atoms, resonant with a cavity mode. Whenever the number of photons in the cavity is such that a single atom from the beam undergoes a full set of Rabi turns while crossing the cavity, the field in the cavity will not evolve. Even though trapping states are rendered unstable by dissipation, the vacuum state is weakly affected at low temperatures, so this state is a preferred candidate for the observation of the trapping effect (which will occur when the vacuum Rabi angle developed by each atom is a multiple of 2π). The presence of a second atom in the cavity would spoil this effect, however. This fact may actually preclude observation of these states, for Poissonian atomic beams.

It was shown in Ref. [8] that collective effects may change dramatically the photon-number steady-state distribution, even when more than 99% of the atoms participate in one-atom events. From an experimental point of view, however, it would be important to know how much time it takes for the trapping state to leak, for a given value of the atomic flux. In the present work, we address the question of the stability of trapping states, calculating the lifetime of the vacuum state as a function of the average number of atoms in the cavity, for a Poissonian beam of resonant two-level atoms, with a mean velocity chosen so that the vacuum is a trapping state. Two methods are used to calculate this lifetime: the Monte Carlo wave-function approach (MCWF), as introduced by Dalibard *et al.* [9], and an approximate analytical solution, motivated by that method. The agreement between the two results is excellent, for a wide range of situations, which include different temperatures, atomic velocity dispersions, and field profiles in the cavity. We do not include here the incoherent atomic relaxation due to stray electrostatic fields [10], which may affect the lifetime of the vacuum state in an important way. Even though this effect can be treated by the Monte Carlo approach, its inclusion requires a careful consideration of the experimental configuration, which is beyond the scope of this paper.

The MCWF simulations involve two steps [9]. In the first one, the Schrödinger equation is numerically integrated from t to $t+\delta t$ with the effective non-Hermitian Hamiltonian $H_{\rm eff}=H(t)-(i\hbar/2)\Sigma_m C_m^\dagger C_m$, where $H(t)$ is the interaction Hamiltonian. We consider for simplicity that the atoms are resonant with a cavity mode, and neglect the atomic decay. Assuming that the atom can be approximated by a two-level system, and adopting the usual electric dipole and rotating wave approximations, it may be written as $H(t)=\Sigma_i \kappa_i(t)\hbar(a\sigma_i^+ + a^\dagger\sigma_i^-)$, where $\kappa_i(t)$ is the coupling constant between atom i and the field, a and $a^\dagger$ are the creation and destruction operators for the cavity mode, and σ_i^+ and σ_i^- are the Pauli spin-flip matrices corresponding to atom i. The number of atoms included in the Hamiltonian for each time interval δt is determined by random choice: before the integration of the Schrödinger equation for each realization, the arrival times of the successive atoms are drafted according to the distribution for time intervals corresponding to a Poissonian pumping [$P(t)=t_{\rm at}^{-1}\exp(-t/t_{\rm at})$, where $t_{\rm at}$ is

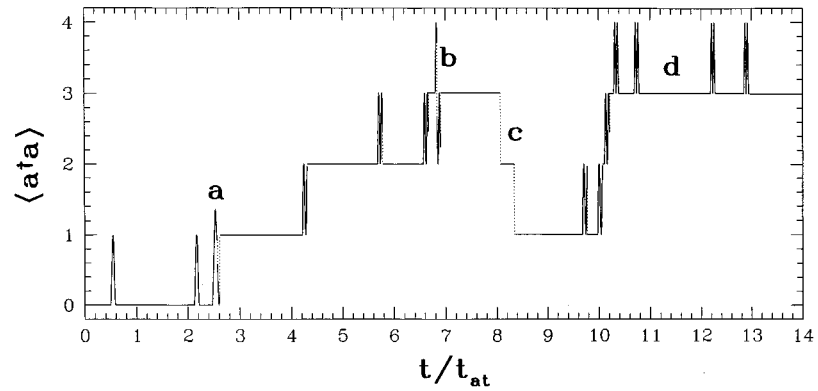

FIG. 1. Single realization of a Poissonian-pumped micromaser with collective effects. The initial state of the field is the vacuum, which is a trapping state. The temperature and the velocity dispersion are zero, the field profile is constant, and the incoming atoms are excited and resonant with the cavity mode. The dashed lines correspond to quantum jumps. Here $N_{ex}=10$ and the average number of atoms in the cavity is $N=0.1$.

the average time interval between successive atoms]. The interaction time between each atom and the cavity mode, which depends on the atomic velocity, is also determined by random choice, previously to the integration of the Schrödinger equation. In the present work, we consider up to two atoms inside the cavity. This restriction is implemented in the following way: whenever the random choice leads to three atoms inside the cavity (this will happen when the arrival times of the first and third atoms differ by a time less than the transit time of the first atom inside the cavity), the third one is delayed. In order not to change the average atomic flux, this delay is compensated by advancing one or more of the following atoms in the sequence, in such a way as to avoid the occupation of the cavity by three or more atoms. One should note, however, that three-atom events are very rare, for the small atomic fluxes considered in the present work. The operators C_m are obtained from the master equation for the reduced density matrix ρ corresponding to the subsystem atoms-field mode (obtained by tracing out the reservoir variables for both the atoms and the field), written in Lindblad's form [11]: $\dot{\rho}=-(i/\hbar)[H,\rho]+\Sigma_m[C_m\rho C_m^\dagger-\frac{1}{2}(C_m^\dagger C_m\rho+\rho C_m^\dagger C_m)]$. The interaction of the field in the cavity with the reservoir is taken into account by the operators $C_1=[\Gamma(1+\overline{n})]^{1/2}a$ and $C_2=[\Gamma\overline{n}]^{1/2}a^\dagger$, where $\Gamma\equiv 1/t_{cav}$ is the decay rate of the field mode and $\overline{n}$ is the average number of thermal photons in the mode, given by Planck's formula.

In the second step, the subsystem is subjected to quantum jumps [9] in each interval δt, according to the probability $\delta\mathcal{P}=\Sigma_m\delta p_m$, where $\delta p_m=\delta t\langle\Psi(t)|C_m^\dagger C_m|\Psi(t)\rangle$. If there is no jump, we have only to normalize the wave function, since the time evolution with H_{eff} is not unitary. If a quantum jump occurs between t and $t+\delta t$, the wave function is projected according to $|\Psi(t+\delta t)\rangle=C_m|\Psi(t)\rangle/(\delta p_m/\delta t)^{1/2}$. The operator C_m to be used in this equation is chosen according to the probability $\delta p_m/\delta\mathcal{P}$. This procedure is repeated $t_{max}/\delta t$ times from $t=0$ to $t=t_{max}$. The expectation value of any operator may be calculated for a single realization at each time interval δt, while the mean value over an ensemble is obtained by making an average over many realizations.

Figure 1 shows the evolution of the number of photons in the cavity for a single realization of the micromaser, as a function of time, for a beam of Poissonian-pumped excited two-level atoms crossing a cavity at zero temperature. The mean number of atoms in the cavity is $N\equiv t_{int}/t_{at}=0.1$, where t_{int} is the atomic transit time through the cavity. The electric field profile is taken to be constant along the cavity and $N_{ex}\equiv t_{cav}/t_{at}=10$. The field starts in the vacuum, which is a trapping state ($\kappa t_{int}=\pi$). Between atoms, the field is in a Fock state, so in this case the figure displays the actual number of photons in the cavity, while when one or more atoms are in the cavity, the expected number of photons is shown. The atoms are measured right after leaving the resonator. During the transit time of one atom a rapid oscillation of the expected number of photons $\langle a^\dagger a\rangle$ can be observed. If during this time there is only one atom in the cavity, the atom emerges completely inverted, since the vacuum is a trapping state. For this simulation the leaking of the photon-number distribution occurs around time $t=2.6t_{at}$ (event *a*), due to a collective event of two atoms. At $t\approx 7t_{at}$ (event *b*) a photon dissipation occurs while one atom is crossing the cavity. Between times $t=8t_{at}$ and $t=9t_{at}$ two free-decaying events can be observed (events *c*). At time $t\approx 10.2t_{at}$ a second trapping state is reached, corresponding to a Rabi angle of 4π (event *d*).

Figure 2 shows the steady-state normalized variance and mean photon number versus the mean number of atoms N in the cavity, for zero temperature, with $N_{ex}=10$ and $\kappa t_{int}=\pi$. We compare our results (obtained by averaging over 2000 realizations) with those obtained in Ref. [8] (con-

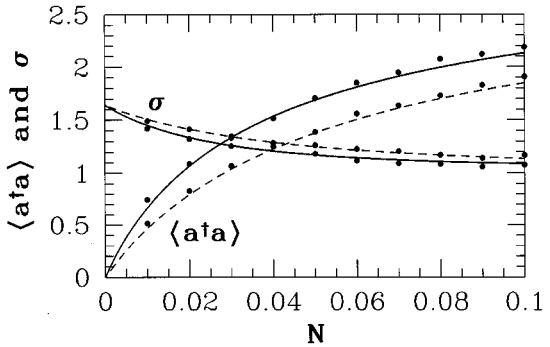

FIG. 2. Steady-state normalized variance σ and average number of photons as a function of the average number of atoms in the cavity, $\sigma\equiv[\langle(a^\dagger a)^2\rangle/\langle a^\dagger a\rangle-\langle a^\dagger a\rangle]^{1/2}$. Atomic vacuum Rabi angle is equal to 2π and $N_{ex}=10$. The Monte Carlo results (dots) are compared with those from Ref. [8], for a constant field profile with $\kappa=\pi/t_{int}$ (full curve). For a sinusoidal field profile, with $\kappa(t)=(\pi^2/2t_{int})\sin(\pi t/t_{int})$, our results match a scaled version of the same curve (dashed curve).

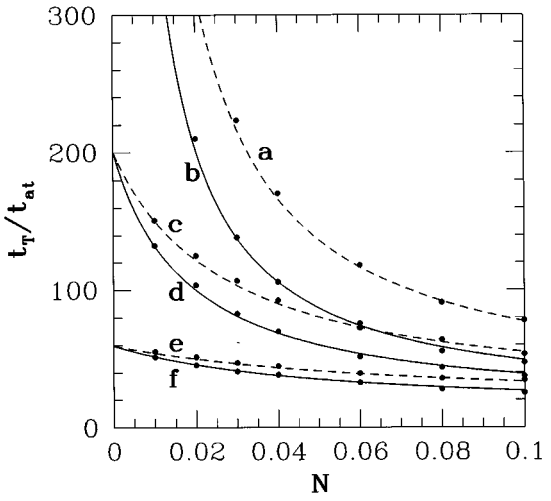

FIG. 3. Lifetime of the vacuum trapping state as a function of the average number of atoms inside the cavity. Atomic vacuum Rabi angle is equal to 2π and $N_{ex}=100$. Dashed lines correspond to sinusoidal field profile (one antinode), while continuous lines stand for constant field profile. (a) and (b) denote zero temperature, monokinetic atomic beam; (c) and (d) denote $\overline{n}=0.5$, monokinetic beam; (e) and (f) denote $\overline{n}=0.5$, and velocities uniformly distributed between $v_T(1-\alpha)$ and $v_T(1+\alpha)$, with $\alpha=0.06$.

tinuous lines), where the collective effects are represented by letting pairs of atoms enter and leave the resonator simultaneously, the decay of the photon field during the passage of the atom pair is neglected, and the field profile is assumed to be constant. Our Monte Carlo results are represented by dots, and correspond to both a constant and a sinusoidal field profile (with just one antinode). The dashed lines correspond to the results of Ref. [8] but with the interaction time replaced by an effective time ($t_{int}\rightarrow 2t_{int}/\pi$), while still keeping $\kappa t_{int}=\pi$, so as to simulate the sinusoidal profile. For the range of parameters here considered, the agreement between the two results is excellent.

The dots in Fig. 3 display the results obtained for the lifetime of the trapping states versus the mean number of atoms N using the MCWF method. The one-atom vacuum-Rabi angle is again taken as 2π. This lifetime is obtained by taking the average of the leaking times over 2000 realizations, starting from the same initial conditions. The continuous lines in the same figure correspond to an analytical approximation, inspired by the MCWF method, and described in the following. The agreement between the two results is excellent.

The analytical approximation is based on the probability δp of having a field transition from the zero-photon state $|0\rangle$ to $|1\rangle$ or $|2\rangle$ during a small time interval δt, which can be written as $\delta p=\delta t/t_T$, where t_T is the lifetime of the trapping state. This probability is calculated by writing it as the sum of three contributions:

$$\delta p=\delta t/t_T=\delta p_r+\delta p_c+\delta p_d, \tag{1}$$

where δp_r is the probability to have a photon excitation in the cavity due to the interaction with the reservoir, δp_c expresses the probability that the trapping state $|0\rangle$ is destroyed by a collective event, and δp_d is the probability that the vacuum state is destroyed by a single-atom event, due to the dispersion in velocities.

We calculate in the following each of the above contributions, up to first order in the average number of atoms in the cavity. The probability δp_r can be approximated by $\delta p_r\approx\Gamma\overline{n}\delta t(1+\overline{\langle a^\dagger a\rangle})$. Note that there is a double average over the photon number in this equation. The brackets refer to the average number of photons between t and $t+\delta t$, while the bar stands for the time average of this quantity (note that even when the field is in the vacuum state between atoms, the average photon number is different from zero while an atom crosses the cavity). For an average number of atoms in the cavity $N\ll 1$ and a constant electric field profile along the resonator,

$$\overline{\langle a^\dagger a\rangle}=\frac{1}{t_{at}}\int_0^{t_{int}}\sin^2(\pi t/t_{int})dt=N/2,$$

where $\sin^2(\pi t/t_{int})$ is the probability that one atom gets deexcited at time t after it enters the cavity, if there is no other atom in the resonator. Therefore, one may write

$$\delta p_r=\Gamma\overline{n}\delta t(1+N/2). \tag{2}$$

Our calculation of δp_c includes only two-atom collective events. It is given by the following expression:

$$\delta p_c=(\delta t/t_{at})(f_1\overline{|g_1|^2}+f_2\overline{|g_2|^2}). \tag{3}$$

The factor $\delta t/t_{at}$ represents the probability to have one atom leaving the cavity during the time interval δt. The factor f_1 (f_2) represents the probability that this emerging atom is the first (second) one of a two-atom event, while $\overline{|g_1|^2}$ stands for the average probability to have the first of the two atoms of a pair emerging from the cavity in the lower resonant state. The quantity $\overline{|g_2|^2}$ is the average conditional probability that the second atom emerges from the cavity in the lower resonant state, if the first one was detected in the upper state. Therefore, Eq. (3) expresses the fact that the trapping state will leak if one of the two atoms is found in the lower state after exiting the cavity. The probabilities f_1 and f_2 are given by [8]: $f_1=f_2=(e^{-2N}-e^{-3N})\equiv f$. The probabilities $\overline{|g_1|^2}$ and $\overline{|g_2|^2}$ are calculated, for a constant field profile, according to the following procedure. The first atom of the pair enters the cavity at time t and leaves it at time $t+t_{int}$. The second atom enters the cavity at time $t+\tau$ and leaves it at time $t+\tau+t_{int}$. For a collective event to occur, one must have $0<\tau<t_{int}$. The Hamiltonian evolution of the atoms-field state (valid for $\Gamma t_{int}\ll 1$) is first calculated from t to $t+t_{int}$. At this time the probability $|g_1(\tau)|^2$ of finding the first atom in the lower state is calculated, and the wave function is projected onto the subspace corresponding to atom one in the upper state, and normalized. This wave function is then evolved from $t+t_{int}$ to $t+\tau+t_{int}$ and the probability $|g_2(\tau)|^2$ of finding the second atom in the lower state is calculated. The mean probability $\beta=\overline{|g_1|^2}+\overline{|g_2|^2}$ is given by

$$\beta=[(1-e^{-N})t_{at}]^{-1}\int_0^{t_{int}}[|g_1(\tau)|^2+|g_2(\tau)|^2]e^{-\tau/t_{at}}d\tau.$$

In zeroth order in N, we find $\beta=0.27$.

Finally, the probability δp_d, associated with deviations from the ideal single-atom trapping-state atomic velocity, is written as the product of the probability $\delta t/t_{\rm at}$ that one atom is leaving the cavity during the time interval δt by the probability $\overline{|g|^2}$ that the emerging atom be found in the lower resonant state when leaving the cavity, averaged over the velocity distribution. We assume that the atoms have a narrow distribution of velocities around the trapping velocity v_T, so that we may take the velocity distribution as constant between $v_-=v_T-\alpha v_T$ and $v_+=v_T+\alpha v_T$, with $\alpha\ll 1$. For an atom with velocity v, the probability of leaving the cavity in the lower state is $|g|^2=\sin^2(\pi v_T/v)$. The average value is thus $\overline{|g|^2}=(1/2v_T\alpha)\int_{v_-}^{v_+}\sin^2(\pi v_T/v)dv=(\pi^2/3)\alpha^2+O(\alpha^4)$, so that $\delta p_d\approx(\delta t/t_{\rm at})(\pi^2/3)\alpha^2$. From this result and from (1), (2), and (3), one gets $\delta t/t_T=(\delta t/t_{\rm at})[t_{\rm at}\Gamma\overline{n}(1+N/2)+f\beta+(\pi^2/3)\alpha^2]$, and therefore

$$\frac{t_{\rm at}}{t_T}=\frac{\overline{n}}{N_{\rm ex}}(1+N/2)+(e^{-2N}-e^{-3N})\beta+\frac{\pi^2}{3}\alpha^2. \qquad (4)$$

This expression is plotted in Fig 3 (continuous curves), using that, for a constant field profile, $\beta=0.27$. For a sinusoidal field profile, we choose the value of β so as to fit the curves to the dots [the best value is $\beta=0.27(2/\pi)\approx 0.17$]. The agreement with the results obtained using the MCWF method is excellent. Figure 3 shows that the lifetime is very sensitive with respect to the average number of atoms in the resonator. For an average number of atoms in the cavity equal to $N=0.1$, with $\overline{n}=0$, $\alpha=0$, and a sinusoidal field profile, the lifetime corresponds to the passage of about 76 atoms through the cavity. This value may decrease to about 54 atoms, if the number of thermal photons is $\overline{n}=0.5$ and $N_{\rm ex}=100$.

In conclusion, our results show that the lifetime of the vacuum trapping state is strongly dependent on atomic collective effects. The application of the Monte Carlo wave-function method in this case allows a realistic simulation of the experimental situation, since it is easy to include effects such as the field profile in the cavity, the atomic velocity distribution, the temperature of the cavity environment, the decay of the field while the atoms are in the cavity, and the atomic decay. Furthermore, we have derived an analytical expression for the lifetime which includes collective effects, and also takes into account finite temperatures and the atomic velocity spread. This expression is in excellent agreement with the Monte Carlo results.

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